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Dynamical Models in Biology
Dr. Z
HW1

1. **$c_n = c_{n-3}*p_3 + c_{n-2}*p_2 + c_{n-1}*p_1$ with initial conditions: $c(n=0)=c_0$, $c(n=1)=c_1$, $c(n=2)=c_2$**
   c_n being the expected number of females born at time n
   females born at n=4: $c(n=4) = c_1*p_3 + c_2*p_2 + c_3*p_1$
   where $c_3 = c(n=3) = c_0*p_3 + c_1*p_2 + c_2*p_1$
   therefore **$c_4 = c(n=4) = c_1*p_3 + c_2*p_2 + (c_0*p_3 + c_1*p_2 + c_2*p_1)*p_1$**
2.

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> #F(n) is the number of females born at time n based on the intial conditions
  # c0 females born at n=0; c1 females born at n=1; c2 females born at n=2;
  # p1 is the probability of a one-year-old to give birth to a new female; similarly for p2 and p3.
  F := proc(c0, c1, c2, p1, p2, p3, n) option remember:
  if n = 0 then
  c0:
  elif n = 1 then
  c1:
  elif n = 2 then
  c2:
  else
  expand(p3·F(c0, c1, c2, p1, p2, p3, n − 3) + p2·F(c0, c1, c2, p1, p2, p3, n − 2) + p1·F(c0, c1, c2, p1, p2, p3, n − 1));
  fi:
  end:
> F(c0, c1, c2, p1, p2, p3, 4)
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$$c0\,p1\,p3 + c1\,p1\,p2 + c2\,p1^2 + p3\,c1 + p2\,c2$$

3. Extinction: (p1,p2,p3)= (.2, .4, .25)

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> seq(F(1, 1, 1, .2, .4, .25, i), i = 950..1000)
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$6.292895169\ 10^{-33}, 5.819778594\ 10^{-33}, 5.382232179\ 10^{-33}, 4.977581666\ 10^{-33}, 4.603353853\ 10^{-33}, 4.257261482\ 10^{-33},$
$3.937189253\ 10^{-33}, 3.641180907\ 10^{-33}, 3.367427252\ 10^{-33}, 3.114255126\ 10^{-33}, 2.880117153\ 10^{-33}, 2.663582294\ 10^{-33},$
$2.463327101\ 10^{-33}, 2.278127626\ 10^{-33}, 2.106851939\ 10^{-33}, 1.948453214\ 10^{-33}, 1.801963325\ 10^{-33}, 1.666486935\ 10^{-33},$
$1.541196021\ 10^{-33}, 1.425324809\ 10^{-33}, 1.318165104\ 10^{-33}, 1.219061950\ 10^{-33}, 1.127409634\ 10^{-33}, 1.042647983\ 10^{-33},$
$9.642589377\ 10^{-34}, 8.917633892\ 10^{-34}, 8.247182487\ 10^{-34}, 7.627137398\ 10^{-34}, 7.053708948\ 10^{-34}, 6.523392371\ 10^{-34},$
$6.032946403\ 10^{-34}, 5.579373466\ 10^{-34}, 5.159901347\ 10^{-34}, 4.771966256\ 10^{-34}, 4.413197156\ 10^{-34}, 4.081401270\ 10^{-34},$
$3.774550680\ 10^{-34}, 3.490769933\ 10^{-34}, 3.228324577\ 10^{-34}, 2.985610558\ 10^{-34}, 2.761144426\ 10^{-34}, 2.553554252\ 10^{-34},$
$2.361571260\ 10^{-34}, 2.184022060\ 10^{-34}, 2.019821479\ 10^{-34}, 1.867965935\ 10^{-34}, 1.727527294\ 10^{-34}, 1.597647203\ 10^{-34},$
$1.477531842\ 10^{-34}, 1.366447073\ 10^{-34}, 1.263713952\ 10^{-34}$

Stability: (p1,p2,p3)= (.353, .33, .329)

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> seq(F(1, 1, 1, .353, .33, .329, i), i = 950..1000)
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308.5444920, 310.4143846, 312.2956093, 314.1882349, 316.0923305, 318.0079657, 319.9352103, 321.8741346, 323.8248096,
325.7873064, 327.7616967, 329.7480524, 331.7464462, 333.7569510, 335.7796401, 337.8145876, 339.8618675,
341.9215547, 343.9937244, 346.0784522, 348.1758142, 350.2858869, 352.4087476, 354.5444735, 356.6931426,
358.8548336, 361.0296252, 363.2175967, 365.4188282, 367.6334000, 369.8613928, 372.1028882, 374.3579677,
376.6267139, 378.9092095, 381.2055380, 383.5157829, 385.8400288, 388.1783606, 390.5308634, 392.8976233,
395.2787265, 397.6742603, 400.0843117, 402.5089689, 404.9483205, 407.4024553, 409.8714633, 412.3554341,
414.8544589, 417.3686287

Explosion: (p1,p2,p3)= (.47, .35, .4)

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> seq(F(1, 1, 1, .47, .35, .4, i), i = 950..1000)
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$9.740446974\ 10^{42}, 1.081171003\ 10^{43}, 1.200079156\ 10^{43}, 1.332064933\ 10^{43}, 1.478566624\ 10^{43}, 1.641180702\ 10^{43},$
$1.821679222\ 10^{43}, 2.022029129\ 10^{43}, 2.244413699\ 10^{43}, 2.491256323\ 10^{43}, 2.765246918\ 10^{43}, 3.069371244\ 10^{43},$
$3.406943435\ 10^{43}, 3.781642116\ 10^{43}, 4.197550495\ 10^{43}, 4.659200848\ 10^{43}, 5.171623918\ 10^{43}, 5.740403736\ 10^{43},$
$6.371738466\ 10^{43}, 7.072507954\ 10^{43}, 7.850348695\ 10^{43}, 8.713737057\ 10^{43}, 9.672081642\ 10^{43}, 1.073582582\ 10^{44},$
$1.191656153\ 10^{44}, 1.322715561\ 10^{44}, 1.468189000\ 10^{44}, 1.629661738\ 10^{44}, 1.808893391\ 10^{44}, 2.007837102\ 10^{44},$
$2.228660820\ 10^{44}, 2.473770927\ 10^{44}, 2.745838464\ 10^{44}, 3.047828230\ 10^{44}, 3.383031101\ 10^{44}, 3.755099883\ 10^{44},$
$4.168089122\ 10^{44}, 4.626499286\ 10^{44}, 5.135325810\ 10^{44}, 5.700113530\ 10^{44}, 6.327017107\ 10^{44}, 7.022868100\ 10^{44},$
$7.795249406\ 10^{44}, 8.652577899\ 10^{44}, 9.604196145\ 10^{44}, 1.066047422\ 10^{45}, 1.183292269\ 10^{45}, 1.313431810\ 10^{45},$
$1.457884214\ 10^{45}, 1.618223622\ 10^{45}, 1.796197301\ 10^{45}$

Stability can be found when: p1 + p2 + p3 = 1 (or is close to equaling 1)

When the probabilities sum to ~ 1, the number of females born at time n is a constant equal to whatever constant c0=c1=c2 are set to. Whether or not this is stable from a biological standpoint is debatable because, in this case, something such as a disease or predator could easily wipe out a population that only births one new female per time n (assuming n is a year). However, in terms of mathematical growth, this would be stable.

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> seq(F(1, 1, 1, .25, .5, .25, i), i = 950 ..1000)
1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000,      (3)
    1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000,
    1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000,
    1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000,
    1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000,
    1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000,
    1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000,
    1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000,
    1.000000000, 1.000000000
```

Population extinction can be found when the probabilities sum to << than 1.
Popilation explosion can be found when the probabilities sum to >> 1.