```
> # Question 1
  # R(n) = p1·R(n-1) + p2·R(n-2) + p3·R(n-3)
  # R(4) = p1·R(3) + p2·R(2) + p3·R(1)
  # R(4) = p1·(p1·R(2) + p2·R(1) + p3·R(0)) +p2·R(2) + p3·R(1)
  # R(4) = p1·p1·c2 + p1·p2·c1 + p1·p3·c0 + p2·c2 + p3·c1
>
  # Question 2

  R := proc(n, p1, p2, p3, c0, c1, c2) option remember :
  if n = 0 then
   c0 :
  elif n = 1 then
   c1 :
  elif n = 2 then
  c2 :
  else
  p1·R(n − 1, p1, p2, p3, c0, c1, c2) + p2·R(n − 2, p1, p2, p3, c0, c1, c2) + p3·R(n − 3, p1,
      p2, p3, c0, c1, c2) :
  fi:
  end:

  R(4, p1, p2, p3, c0, c1, c2);      #shows to confirm specific result from Question 1
```

$$p1\ (p3\ c0 + p2\ c1 + p1\ c2) + p2\ c2 + p3\ c1 \qquad \textbf{(1)}$$

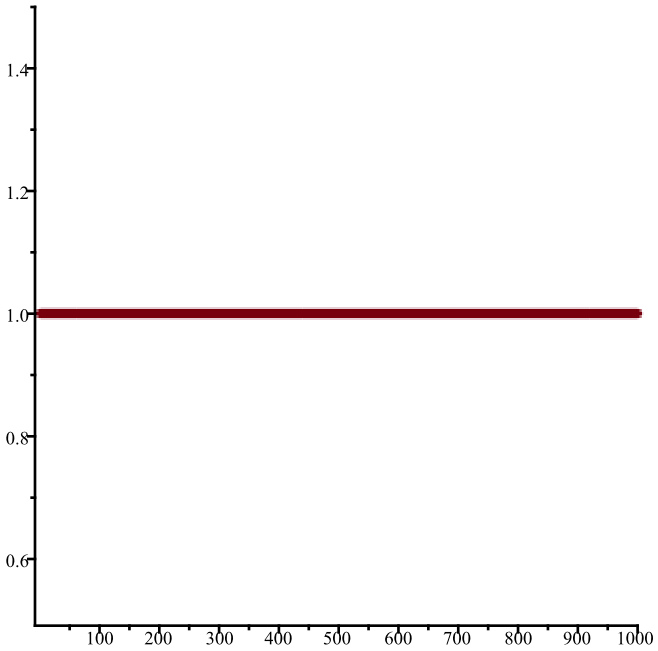
```
> # Question 3
  # assume c0=c1=c2
  # at time n=1000, stable, extinction, explosion.0

  # 1st lesson from experimenting, when p1+p2+p3 = 1, there will be stability
```

that is, when for example p1=p2=p3= . $\frac{1}{3}$,

or p1=. $\frac{1}{4}$ **and** $p2 = \frac{1}{4}$ **and** $p3 = \frac{1}{2}$, *etcetera* ....

$$plot\left([seq]\left(\left[i, R\left(i, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 1, 1, 1\right)\right], i = 1..1000\right), style = point\right);$$



> # 2nd lesson from experimenting, when $p1+p2+p3>1$, there will be explosion

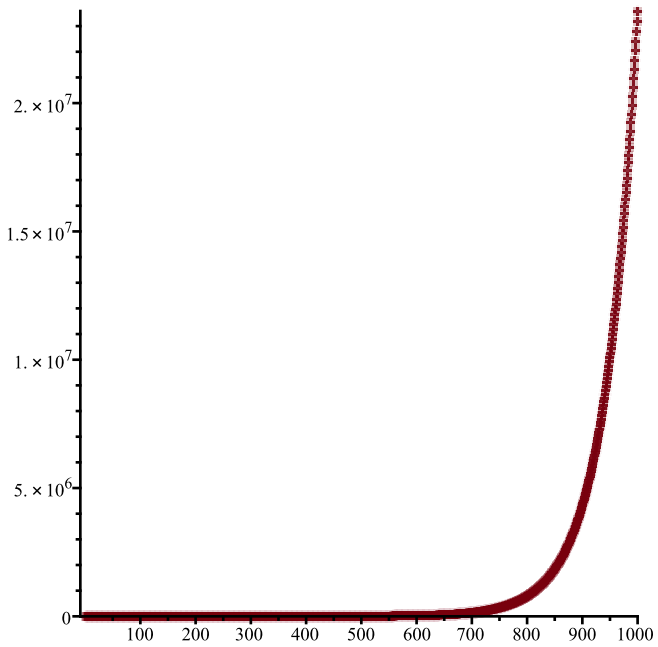
example: $p1=p2=p3=\frac{1}{2.9}$,

or $p1 = \frac{1}{3.9}$ **and** $p2 = \frac{1}{3.9}$ **and** $p3 = \frac{1}{1.9}$

On this specific timescale(n= 1000), it is very difficult to find p values which do not show a strong upward exponential trend

One further conclusion is that as p values increase, so increases the time before the most dramatic exponential turn(respective of this scale), the further the distance between the sum $p1 + p2 + p3$ **and** 1, the earlier the mass of exponential growth is seen

$$plot\left([seq]\left(\left[i, R\left(i, \frac{1}{2.9}, \frac{1}{2.9}, \frac{1}{2.9}, 1, 1, 1 \right) \right], i = 1..1000 \right), style = point \right);$$



> # The third lesson is that when $p1+p2+p3<1$ there will be exponential downturn, ie. extinction

for example $p1=p2=p3=\frac{1}{3.1}$

or $p1=\frac{1}{4.1}$ **and** $p2=\frac{1}{4.1}$ **and** $p3=\frac{1}{2.1}$

the larger the difference between the sum $p1+p2+p3$ and 1 the earlier the downturn

$$plot\left([seq]\left(\left[i, R\left(i, \frac{1}{3.1}, \frac{1}{3.1}, \frac{1}{3.1}, 1, 1, 1\right)\right], i=1..1000\right), style=point\right);$$