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Math 336

1. In a certain species of animals, only one-year-old, two-year-old, and three-year-old females are fertile.

The probabilities of a one-year-old, two-year-old, and three-year-old female to give birth to a new female are p_1, p_2, p_3 respectively. 1. In a certain species of animals, only one-year-old, two-year-old, and three-year-old females are fertile.

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Assuming that there were c_0 females born at n = 0, c_1 females born at n = 1, and c_2 females born at n = 2. Set up a recurrence that will enable you to find the expected number of females born at time n.

In terms of c_0, c_1, c_2, p_0, p_1, p_2, how many females were born at n = 4?

$B(n) = p_1B(n-1) + p_2B(n-2) + p_3B(n-3)$

$B(3) = p_1 c_2 + p_2c_1 + p_3c_0$

$B(4) = p_1B(3) + p_2B(2) + p_3B(1)$

$= p_1(p_1 c_2 + p_2c_1 + p_3c_0) + p_2c_2 + p_3c_1$

$= (p_1^2 + p_2) c_2 + (p_1p_2 + p_3)c_1 + p_1 p_3 c_0$

2. Write the Maple code F (c0, c1, c2, p1, p2, p3, n) that would input n (the discrete time) and

output the number of females born at time n.

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F := proc(n, p1, p2, p3, c0, c1, c2) option remember:

if n = 0 then
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c0:

elif n = 1 then

c1:

elif n = 2 then

c2:

else

expand(p1*F(n - 1, p1, p2, p3, c0, c1, c2) + p2*F(n - 2, p1, p2, p3, c0, c1, c2) + p3*F(n - 3, p1, p2, p3,
c0, c1, c2)):

fi:

end:
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3. Taking $c_0 = c_1 = c_2 = 1$, experiment with values of p_1, p_2, p_3 that would lead, at time n = 1000, to (i) extinction (ii) stable population (iii) population explosion.

(i) When $p_1 + p_2 + p_3 < 1$, the population deteriorates. As an example, if $p_1 = p_2 = p_3 = 0$, the population will be extinct at time n= 1000. Or if $p_1 = p_2 = p_3 = 1/18$, there will be no rabbits left at n=1000.

(ii) When $p_1 + p_2 + p_3 =1$, the population stabilizes.

(iii) When $p_1 + p_2 + p_3 > 1$, the population explodes. For example, if $p_1 = p_2 = p_3 = 1$, there will be $1.9*10^{264}$ rabbits when n = 1000.