

> question 1. In a certain species of animals, only one-year-old, two-year-old, and three-year-old females are fertile.

The probabilities of a one-year-old, two-year-old, and three-year-old female to give birth to a new female are  $p_1$ ,  $p_2$ ,  $p_3$  respectively.

Assuming that there were  $c_0$  females born at  $n = 0$ ,  $c_1$  females born at  $n = 1$ , and  $c_2$  females born at  $n = 2$ . Set up a recurrence that will enable you to find the expected number of females born at time  $n$

$$\text{Answer: } C_n = P_3(C_{n-3}) + P_2(C_{n-2}) + P_1(C_{n-1})$$

$$C_3 = P_3(C_0) + P_2(C_1) + P_1(C_2)$$

$$C_4 = P_3(C_1) + P_2(C_2) + P_1(C_3)$$

$$C_4 = P_3(C_1) + P_2(C_2) + P_1(P_3(C_0) + P_2(C_1) + P_1(C_2))$$

question 2. Write the Maple code  $F(c_0, c_1, c_2, p_1, p_2, p_3, n)$  that would input  $n$  (the discrete time) and output the number of females born at time  $n$  (see code below)

question 3. Taking  $c_0 = c_1 = c_2 = 1$ , experiment with values of  $p_1, p_2, p_3$  that would lead, at time  $n = 1000$ ,

to (i) extinction (ii) stable population (iii) population explosion. (see code below)

Stable population occurs when  $p_1 = p_2 = p_3 = .33333$ . Extinction occurs  $p_1 = p_2 = p_3 < .33333$  and population explosion occurs when  $p_1 = p_2 = p_3 > .33333$

$$p_1 := 1$$

(1)

>  $Rg := \text{proc}(n, p_1, p_2, c_0, c_1) \text{ option remember} :$

**if**  $n = 0$  **then**

$c_0 :$

**elif**  $n = 1$  **then**  $c_1 :$

**else**

$\text{expand}(p_1 \cdot Rg(n - 1, p_1, p_2, c_0, c_1) + p_2 \cdot Rg(n - 2, p_1, p_2, c_0, c_1)) :$

**fi:**

**end:**

>  $\text{seq}(Rg(i, .5, 1, 1, 1), i = 1 .. 10);$

1, 1.5, 1.75, 2.375, 2.9375, 3.84375, 4.859375, 6.2734375, 7.99609375, 10.27148438

(2)

>  $Z := \text{proc}(n, p_1, p_2, p_3, c_0, c_1, c_2) \text{ option remember} :$

**if**  $n = 0$  **then**

$c_0 :$

**elif**  $n = 1$  **then**  $c_1 :$

**elif**  $n = 2$  **then**  $c_2 :$

**else**

$\text{expand}(p_1 \cdot Z(n - 1, p_1, p_2, p_3, c_0, c_1, c_2) + p_2 \cdot Z(n - 2, p_1, p_2, p_3, c_0, c_1, c_2) + p_3 \cdot Z(n - 3, p_1, p_2, p_3, c_0, c_1, c_2)) :$

**fi:**

**end:**

>  $\text{seq}(Z(i, 1, 1, 1, 1, 1, 1), i = 1 .. 10);$

1, 1, 3, 5, 9, 17, 31, 57, 105, 193

(3)

>  $\text{seq}(Z(i, .33333, .33333, .33333, 1, 1, 1), i = 1 .. 1000)$

1, 1, 0.99999, 0.9999866667, 0.9999822223, 0.9999762965, 0.9999717287, 0.9999667494, 0.9999615918, 0.9999566903, 0.9999516776, 0.9999466537, 0.9999416744,

(4)

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