Timothy Nosralla Final Exam
Due 12/14/21

1) · Number of rabbits at a day = 2· number of rabbits yesterday − number of rabbits 3 days ago

$r(n) = 2\cdot r(n-1) - r(n-3)$ $\qquad r(0)=1 \quad r(1)=1 \quad r(2)=2$

$\rightarrow z = 2\cdot z[1] - z[3]$

To evaluate the number of rabbits at day 999 & 1000
I chose to use the OrbK function in DMB.txt using the above transformation.

$r := \text{OrbK}(3, z, 2\cdot z[1] - z[3], [1,1,2], 999, 1000)$

$\text{evalf}(r[2]/r[1]) = \text{Day}1000/\text{Day}999 = 1.618033989 \Rightarrow$ Golden Ratio

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2) Rate of change → Continuous time

$$x'(t) = \underbrace{\frac{5}{2}\cdot x(t)\cdot(1-x(t))\cdot\left(1-\frac{1}{2}x(t)\right)}_{f(x(t))} = \frac{5}{4}(x(t))^3 - \frac{15}{4}(x(t))^2 + \frac{5}{2}x(t)$$

a) Find all equilibrium solutions

Let $x(t)=c$, so $x'(t)=0$

$$0 = \frac{5}{2}\cdot x(t)\cdot(1-x(t))\cdot\left(1-\frac{1}{2}x(t)\right)$$

Solutions: $x(t)=0,\ x(t)=1,\ x(t)=2$

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b) Find all stable equilibrium solutions $\quad f'(x) = \frac{15}{4}x^2 - \frac{15}{2}x + 5/2$

$f'(0) = 5/2$ $\qquad f'(2) = 15 - 15 + 5/2 = 5/2$ $\qquad$ Stable Equi Points: $x(t)=1$

$f'(1) = \frac{15}{4} - \frac{15}{2} + \frac{5}{2} = -1.25$

An equilibrium solution is stable $\Leftrightarrow f'(c) < 0$

c) At time 0, $x(t) = 0.1$. What is the value at $t = 100$.

Since this is a continuous time system, I used dsolve to solve for $x(t)$ given $x(0) = 0.1$, then substituted $t = 100$

dsolve({diff(x(t), t) = 5/2 · x(t) · (1 − x(t)) · (1 − 0.5 · x(t)), x(0) = 0.1}, x(t))

subs (t = 100, %)

$$\underline{x(100) = 0.9999999999}$$

3) Quantity today ⇒ Discrete time

$$x(n) = \underbrace{\frac{5}{2} \cdot x(n-1) \cdot (1 - x(n-1)) \cdot (1 - 0.5 \cdot x(n-1))}_{f(x(n-1))} = \frac{5}{4}(x(n-1))^3 - \frac{15}{4}(x(n-1))^2 + \frac{5}{2} \cdot x(n-1)$$

a) Find all equilibrium solutions

Let $c = f(c)$, solve for c

$$c = \frac{5}{4}c^3 - \frac{15}{4}c^2 + \frac{5}{2}c \to 0 = \overset{c=0}{c}\left(\frac{5}{4}c^2 - \frac{15}{4}c + \frac{3}{2}\right)$$

15/2 → 120/8

$$c = \frac{15/4 \pm \sqrt{\frac{225}{16} - 4 \cdot \frac{5}{4} \cdot \frac{3}{2}}}{(5/2)} = \frac{15}{10} \pm \frac{\sqrt{105/16}}{5/2} \to c = \frac{15}{10} \pm \frac{\sqrt{105}}{10}$$

Sol: $\underline{x(n) = 0 \quad x(n) = \frac{15}{10} + \frac{\sqrt{105}}{10} \quad x(n) = \frac{15}{10} - \frac{\sqrt{105}}{10}}$

b) Find all stable equilibrium solutions

Stable sol $\iff |f'(c)| < 1$

$\iff -1 < f'(c) < 1$

$$f'(c) = 3.75c^2 - 7.5c + 2.5$$

$f'(0) = 2.5$

$f'\left(\frac{15}{10} + \frac{\sqrt{105}}{10}\right) = 7.467$

$f'\left(\frac{15}{10} - \frac{\sqrt{105}}{10}\right) = -0.2178$

$\underline{x(n) = \frac{15}{10} - \frac{\sqrt{105}}{10} \text{ is a stable equilibrium solution}}$

c) If at day 0 its value is 0.1, what would its value be at day 1000?

- Will most likely approach stable equilibrium point

To make sure, OrbF([$\frac{5}{2}\cdot x\cdot(1-x)\cdot(1-0.5\cdot x)$], [x], [0.1], 999, 1000) [2];

yields $0.4753049232 = \frac{15}{10} - \frac{\sqrt{105}}{10}$

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4)

		AA	Aa	aa
		u	v	w
AA	u	u^2 for AA	$\frac{1}{2}uv$ AA, $\frac{1}{2}uv$ Aa ($\times 2$)	uw Aa
Aa	v	$\frac{1}{2}uv$ AA, $\frac{1}{2}uv$ Aa	$\frac{1}{4}v^2$ AA, $\frac{1}{4}v^2$ aa, $\frac{1}{2}v^2$ Aa	$\frac{1}{2}vw$ Aa, $\frac{1}{2}vw$ aa
aa	w	uw Aa	$\frac{1}{2}vw$ Aa, $\frac{1}{2}vw$ aa	aa w^2

$u \to u^2 + uv + \frac{1}{4}v^2$

$v \to u\cdot v + 2\cdot u\cdot w + \frac{1}{2}v^2 + v\cdot w$.

$w \to \frac{1}{4}v^2 + v\cdot w + w^2$

$u, v, \& w$ initially equal $\frac{1}{3}$ each

a) 2nd generation

$$v(\text{\# of genotype \#1}) = \frac{1}{3}\cdot\frac{1}{3} + 2\cdot\frac{1}{3}\cdot\frac{1}{3} + \frac{1}{2}\cdot\frac{1}{9} + \frac{1}{9} = \frac{1}{2}$$

$\frac{1}{2}$ of the second generation has genotype Aa

b) The HW Law dictates that after 1 generation, the Hardy Weinberg equilibrium is attained. Therefore, v at the 2nd generation is the same as the v for all $n \geq 2$

Therefore, the proportion applies. $v = \frac{1}{2}$ for the 1000th generation.

5) Similar to question 4, we want to find the matrix that represents the potential offspring genotypes. However, we are breaking the assumption of random mating. Therefore, I will multiply each value in the Matrix by a value that represents their mating probability - (therefore AA females x Aa males will have the value 2, while the rest will have 1). Then, I will normalize the function by dividing by the sum, since this new sum will not be equal to 1 as per the HW Law. Then, I will apply the Orb function for long term behavior.

Male \ Female	AA u	Aa v	aa w
AA u	$u^2 \cdot .5$ AA	$\boxed{2}$ $.5 \cdot u \cdot v \cdot 1$ Aa $.5 \cdot u \cdot v \cdot 1$ AA	$u \cdot w \cdot .5$ Aa
Aa v	$.5 \cdot u \cdot v \cdot .5$ Aa $.5 \cdot u \cdot v \cdot .5$ AA	$1/4 v^2 \cdot .5$ AA $\frac{1}{2} v^2 \cdot .5$ Aa $\frac{1}{4} v^2 \cdot .5$ aa	$.5 \cdot v \cdot w \cdot .5$ Aa $.5 \cdot v \cdot w \cdot .5$ aa
aa w	$u \cdot w \cdot .5$ Aa	$.5 \cdot v \cdot w \cdot .5$ Aa $.5 \cdot v \cdot w \cdot .5$ aa	aa $\cdot .5 w^2$

ΣAA
$u \to .5 \cdot u^2 + .75 uv + \frac{1}{8} v^2$

ΣAa
$v \to .75 uv + u \cdot w + \frac{1}{4} v^2 + .5 vw$

Σaa
$w \to .5 w^2 + .5 v \cdot w + \frac{1}{8} v^2$

$\text{sum} := .5u^2 + 1.5uv + .5v^2 + u \cdot w + v \cdot w + .5 w^2$

a) $\text{Orb}\left(\left[\frac{0.5 \cdot u^2 + 0.75 \cdot u \cdot v + 0.125 \cdot v^2}{\text{sum}}, \frac{0.75 u \cdot v + u \cdot w + \frac{1}{4} \cdot v^2 + .5 \cdot v \cdot w}{\text{sum}}, \frac{.5 w^2 + 0.5 \cdot v \cdot w + 0.125 \cdot v^2}{\text{sum}}\right], [u, v, w], [1/3, 1/3, 1/3], 0, 1\right)[2][2] = \underline{0.50}$

At the 2nd generation, the proportion of Aa genotypes is $\underline{.50}$

b) $\text{Orb}\left(\left[\frac{0.5 \cdot u^2 + 0.75 \cdot u \cdot v + 0.125 \cdot v^2}{\text{sum}}, \frac{0.75 \cdot u \cdot v + u \cdot w + 0.25 v^2 + .5 \cdot v \cdot w}{\text{sum}}, \frac{.5 w^2 + 0.5 \cdot v \cdot w + 0.125 \cdot v^2}{\text{sum}}\right], [u, v, w], [1/3, 1/3, 1/3], 998, 999\right)[2][2] = \underline{0.3974661807}$

Since this function starts at 0, the 1000th generation is $K_2 = 999$

At the 1000th generation, the proportion of Aa is approximately

6) Since the question asks for y at a very large n, I first checked if there are any stable fixed points

$$x(n) = \underbrace{\frac{1 + x(n-1) + y(n-1)}{2 + x(n-1) + 3 \cdot y(n-1)}}_{f(x(n-1), y(n-1))} \qquad y(n) = \underbrace{\frac{1 + x(n-1) + 3y(n-1)}{2 + x(n-1) + 2y(n-1)}}_{g(x(n-1), y(n-1))}$$

$$x = f(x,y) \qquad y = g(x,y)$$

$$2x + x^2 + 3yx = 1 + x + y \qquad 2y + 2xy + 2y^2 = 1 + x + 3y$$

$$x^2 + x - y + 3 \cdot y \cdot x = 1 \qquad 2y^2 + 2xy - y - x = 1$$

SFP([(1+x+y)/(2+x+3y), (1+x+3y)/(2+x+2y)], [x,y]) yielded $x = 0.4494897428$, $y = 1$

Therefore, y(100000...000.) would be approximately 1.000000000

7i a) To find the long term behavior of the SIRS, I used the SIRS function to make each respective situation into a set of functions, then used Time Series to see the long term behavior

a) x := (SIRS(s, i, 0.05, 0.5, 100, 1000)

Time Series(x, [s,i], [300, 300], 0.01, 10, 1);

Time Series (x, [s,i], [300, 300], 0.01, 10, 2);

shows 0 removed individuals after the initial 400, which implies a full recovery.

7b) y := SIRS(s, i, 1.4, 0.5, 100, 1000)

TimeSeries(y, [s,i], [300, 300], 0.01, 10, 1);

TimeSeries(y, [s,i], [300,300], 0.01, 10, 2);

The result of this is that everyone in this community is removed and therefore dead

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7c) To find the value at which there are no more infections, we must find the R_0 factor

$R_0 = \frac{N \cdot B}{V} = \frac{1000 \cdot B}{100}$. We want this to be less than 1, so the infectious contact rate wont allow for spread.

$\frac{1000 \cdot B}{100} > 1, \quad B < 1/10.$

Therefore the "cut off" for a non-zero number of infected is $B = 1/10$.

8).

a) GeneNet (9, 1, 0.2, 2, m1, m2, m3, p1, p2, p3)

SEquP(0/0, [m1, m2, m3, p1, p2, p3]);

The height is 0.6823278038

b) With a=3, we do have a horizontal asymptote. However, it moved to 1.2134166

c) After testing multiple a parameters, starting at 25 then incrementing downwards, the a value I found was a = 7.39.

When a = 7.4, the SEquP function returned the empty set.

9) ChemoStat(N, C, a1, a2)

$\uparrow$ Bacterial Population

$\leftarrow$ Nutrient Concentration

First I used PhaseDiag to see if the N&C values approached a certain pt

PhaseDiag(Chemostat(N,C,2.5,2.7), [N,C], [1,1], 0.01, 10).

Then I used SEquP after confirming there was a point to get the exact Points

SEquP(Chemostat(N,C,2.5,2.7), [N,C]) $\Rightarrow$

which yielded $[5.0833\overline{3}, 0.166\overline{6}]$

a) The Bacterial population density after a very long time would be 5.0833 cells per liter

b) The Nutrient concentration after a very long time would be 0.1666 units.

10 First, To find the long Term behavior of the mini-web, I first made the matrix that represents the Web page, then raise it to a very large power in Maple which would approximate the behavior in the long term i.e $n=1000$

$$A = \begin{bmatrix} 0.2 & 0.1 & 0.1 & 0.075 & 0.075 & 0.075 & .05 & .05 & .05 \\ 0.1 & 0.2 & 0.1 & 0.075 & 0.075 & 0.075 & .05 & .05 & .05 \\ 0.1 & 0.1 & 0.2 & 0.075 & 0.075 & 0.075 & .05 & .05 & .05 \\ 0.1 & 0.1 & 0.1 & 0.4 & 0.075 & .075 & .05 & .05 & .05 \\ 0.1 & 0.1 & 0.1 & 0.075 & 0.4 & .075 & .05 & .05 & .05 \\ 0.1 & 0.1 & 0.1 & 0.075 & 0.075 & 0.4 & .05 & .05 & .05 \\ 0.1 & 0.1 & 0.1 & 0.075 & 0.075 & 0.075 & .6 & .05 & .05 \\ 0.1 & 0.1 & 0.1 & 0.075 & 0.075 & 0.075 & .05 & .6 & .05 \\ 0.1 & 0.1 & 0.1 & 0.075 & 0.075 & 0.075 & .05 & .05 & .6 \end{bmatrix}$$

The long run is $G := A^{1000}$

Each row i would represent the probability of staying on the i-th site

a) The approximate probability of a random surfer being at webpage 1 is 0.07692307692307801

b) The approximate probability of a random surfer being on web-page 9 is 0.1538461538461560