Note: Updated Maple takes 2D form $\frac{a}{b}$ rather than a/b

Final

1. $x(n) = 2x(n-1) - x(n-3)$, $x(0)=1$, $x(1)=1$, $x(2)=2$

Use Orbk to find long term behavior of 3^{rd} order system

$$\text{evalf}\left(\frac{\text{Orbk}(3, z, 2\cdot z[1]-z[3], [1,1,2], 1000, 1001)[2]}{\text{Orbk}(3, z, 2\cdot z[1]-z[3], [1,1,2], 1000, 1001)[1]}\right)$$ ← Orbk sets the value of $x(0)$ at $k=1$

$= 1.618033989$

2. $x'(t) = \frac{5}{2}x(t)(1-x(t))(1-\frac{1}{2}x(t))$

a) Find underlying function

$f(x) = \frac{5}{2}x(1-x)(1-\frac{1}{2}x)$

By hand: solve where rate of change is 0

$0 = \frac{5}{2}x(1-x)(1-\frac{1}{2}x)$

$x=0,\ x=1,\ x=2$

In Maple: EquP($[\frac{5}{2}\cdot x\cdot(1-x)\cdot(1-\frac{1}{2}\cdot x)], [x]$) ⇒ {[0], [1], [2]}

b) Find derivative of underlying function

$f(x) = (\frac{5}{2}x - \frac{5}{2}x^2)(1-\frac{1}{2}x)$

$= \frac{5}{2}x - \frac{5}{4}x^2 - \frac{5}{2}x^2 + \frac{5}{4}x^3$

$f(x) = \frac{5}{2}x - \frac{15}{4}x^2 + \frac{5}{4}x^3$

$f'(x) = \frac{5}{2} - \frac{15}{2}x + \frac{15}{4}x^2$

Check derivative values at equilibrium points

$f'(0) = \frac{5}{2}$ ⇒ unstable $\frac{5}{2} > 0$

$f'(1) = -\frac{5}{4}$ ⇒ stable $-\frac{5}{4} < 0$ ⇒ $x=1$ is the stable equilibrium point

$f'(2) = \frac{5}{2}$ ⇒ unstable $\frac{5}{2} > 0$

In Maple: SEquP($[\frac{5}{2}\cdot x\cdot(1-x)\cdot(1-\frac{1}{2}\cdot x)][x]$) ⇒ {[1]}

c) $x(100) = 1$ because 1 is a stable equilibrium therefore at initial values around 1, the system will converge to 1

Checked in Maple:

evalf(subs(t=100, dsolve({diff(x(t),t) = $\frac{5}{2}\cdot x(t)\cdot(1-x(t))\cdot(1-\frac{1}{2}\cdot x(t))$, x(0)=0.1}, x(t))))

⇒ $x(100) = 0.9999999999$ ⇒ approximately 1

3. $x(n) = \frac{5}{2}x(n-1)(1-x(n-1))(1-\frac{1}{2}x(n-1))$

a) Find fixed points of underlying function (where $x = F(x)$)

$F = \frac{5}{2}x(1-x)(1-\frac{1}{2}x)$

$x = \frac{5}{2}x(1-x)(1-\frac{1}{2}x)$

Maple: evalf(FP([$\frac{5}{2}\cdot x\cdot(1-x)\cdot(1-\frac{1}{2}\cdot x)$],[x])) ⇒ {[0], [0.4753049230], [2.524695077]}

$x = 0$, $x = 0.475304923$, $x = 2.524695077$

b) Find which equilibrium solution satisfies $|F'(c)| < 1$

$F' = \frac{5}{2} - \frac{15}{2}x + \frac{15}{4}x^2$

$F'(0) = \frac{5}{2}$ ⇒ unstable $|\frac{5}{2}| > 1$

$F'(0.475304923) = -0.217606535645$ ⇒ stable $|-0.217606535645| < 1$

$F'(2.524695077) = 7.46760654186$ ⇒ unstable $|7.46760654186| > 1$

In Maple: evalf(SFP([$\frac{5}{2}\cdot x\cdot(1-x)\cdot(1-\frac{1}{2}\cdot x)$], [x]) ⇒ [[0.4753049232]]

$x = 0.4753049232$ is the stable equilibrium

c) $x(1000) = 0.4753049232$ because the system will converge to the stable equilibrium if it begins in the neighborhood of that equilibrium.

Checked in Maple: Orb([$\frac{5}{2}\cdot x\cdot(1-x)\cdot(1-\frac{1}{2}\cdot x)$], [x], [0.1], 1000, 1000)[1] ⇒ [0.4753049232]

4. If there are equal proportions of AA, Aa, and aa, then $u = v = w = \frac{1}{3}$ because $u + v + w = 1$.

↙ Generation 1 is $k = 0$

a) Orb(HW3(u, v, w), [u, v, w], [$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$], 1, 1)[1][2]

⇒ $\frac{1}{2}$ ⇒ genotype Aa proportion $= \frac{1}{2}$

b) Orb(HW3(u, v, w), [u, v, w], [$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$], 999, 999)[1][2]

⇒ $\frac{1}{2}$ ⇒ genotype Aa proportion $= \frac{1}{2}$

5. If the probabilities of mating are unequal then you must incorporate the probability matrix and multiply each mating with the probability of that match occurring

a) Orb(HW3g (u, v, w, [[.1, .2, .1], [.1, .1, .1], [.1, .1, .1]]), [u, v, w], [$\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$], 1, 1) [1] [2]

⇒ 0.5 ⇒ proportion of genotype Aa = 0.5

Used any probability matrix where AA (female) × Aa (male) was double that of the other crosses

b) Orb(HW3g (u, v, w, [[.1, .2, .1], [.1, .1, .1], [.1, .1, .1]]), [u, v, w], [$\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$], 999, 999) [1] [2]

⇒ 0.3974661806 ⇒ proportion of genotype Aa = 0.3974661806

6. Use underlying transformation to examine long term behavior

$(x, y) \rightarrow \left(\frac{1+x+y}{2+x+3y}, \frac{1+x+3y}{3+x+2y} \right)$

OrbF([$\frac{1+x+y}{2+x+3\cdot y}$, $\frac{1+x+3\cdot y}{3+x+2\cdot y}$], [x, y], [100, 1000], 1000, 1001] [1] [2] ⇒ 0.7478789080

OrbF([$\frac{1+x+y}{2+x+3\cdot y}$, $\frac{1+x+3\cdot y}{3+x+2\cdot y}$], [x, y], [100, 1000], 1000, 1001] [2] [2] ⇒ 0.7478789080

Equilibrium of y at 0.7478789080

7. Find how many susceptible and infected in the long run and then do N − s − i to find recovered (1000 − s − i)

a) SEquP(SIRS(s, i, 0.05, 0.5, 100, 1000), [s, i]) ⇒ {[1000, 0]}

OR Dis(SIRS(s, i, 0.05, 0.5, 100, 1000), [s, i], [300, 300], 0.01, 20) [-1]

⇒ [20.01, [999.9666301, $6.997984915 \times 10^{-686}$]] ⇒ approx. [1000, 0]

1000 − 1000 − 0 = 0 recovered

b) SEquP(SIRS(s, i, 1.4, 0.5, 100, 1000), [s, i]) ⇒ {[71.42857143, 4.619758351]}

1000 − 71.42857143 − 4.619758351 = 923.951670219 recovered

c) Find value of b where i = 0 ⇒ b = 0.1

in Maple:
- SIRS(s, i, b, 0.5, 100, 1000)
- SysEq := {−sib + 500.0 − 0.5s − 0.5i = 0, sib − 100i = 0} ← from SIRS model
- solve for b

8. a) SEquP(GeneNet(0, 1, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])
Stable equilibrium at 0.6823278038

b) TimeSeries(GeneNet(0, 3, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3], [0.2, 0.1, 0.3, 0.1, 0.4, 0.5], 0.01, 200, 1)
⇒ graph in Maple ⇒ there is a horizontal asymptote
SEquP(GeneNet(0, 3, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])
Height of horizontal asymptote = stable equilibrium at 1.213411663

c) Found where the system no longer had a stable equilibrium
First found wher SEquP outputted an empty set with step size 1 between 3 and 50. This gave a = 8 as the least entry with an empty set. Then found where the output was the empty set between 7 and 8 with step size 0.1: 7.4. Then used step size 0.01 and the greatest a with a nonempty set was 7.39. Code in Maple.

9. Found stable equilibrium point of Chemostat model
SEquP(ChemoStat(N, C, 2.5, 2.7), [N, C]) ⇒ {[5.083333333, 0.6666666667]}
(5.083333333: Bacterial pop density; 0.6666666667: Nutrient concentration)
a) Bacterial population density = 5.083333333
b) Nutrient concentration = 0.6666666667

10. Created the transition matrix for the probability of moving from page to page
Rows of transition matrix must sum to 1
Called it matrix P in Maple
Take the limit: $\lim_{n\to\infty} P^n$ or in Maple P^{1000}
Full code in Maple
Site 1: 0.07692307655 Site 9: 0.1538461530

> *#Shreya Ghosh, Maple Code for Final Exam*

```
> read "/Users/shreyaghosh/Documents/DMB.txt"
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

> *#1)*

$$> \ evalf\left(\frac{Orbk(3, z, 2\cdot z[1] - z[3], [1, 1, 2], 1000, 1001)[2]}{Orbk(3, z, 2\cdot z[1] - z[3], [1, 1, 2], 1000, 1001)[1]}\right)$$

$$1.618033989 \quad \textbf{(2)}$$

>

> *#2a)*

$$> \ EquP\left(\left[\frac{5}{2}\cdot x\cdot(1-x)\cdot\left(1-\frac{1}{2}\cdot x\right)\right], [x]\right)$$

$$\{[0], [1], [2]\} \quad \textbf{(3)}$$

> #2b)

> $SEquP\left(\left[\frac{5}{2}\cdot x\cdot(1-x)\cdot\left(1-\frac{1}{2}\cdot x\right)\right],[x]\right)$

$$\{[1.]\} \quad (4)$$

> #2c)

> $evalf\left(subs\left(t=100, dsolve\left(\left\{diff(x(t),t)=\frac{5}{2}\cdot x(t)\cdot(1-x(t))\cdot\left(1-\frac{1}{2}\cdot x(t)\right), x(0)=0.1\right\}, x(t)\right)\right)\right)$

$$x(100)=0.9999999999 \quad (5)$$

>

> #3a)

> $evalf\left(FP\left(\left[\frac{5}{2}\cdot x\cdot(1-x)\cdot\left(1-\frac{1}{2}\cdot x\right)\right],[x]\right)\right)$

$$\{[0.],[0.475304923],[2.524695077]\} \quad (6)$$

> #3b)

> $evalf\left(SFP\left(\left[\frac{5}{2}\cdot x\cdot(1-x)\cdot\left(1-\frac{1}{2}\cdot x\right)\right],[x]\right)\right)$

$$\{[0.475304923]\} \quad (7)$$

> #3c)

> $Orb\left(\left[\frac{5}{2}\cdot x\cdot(1-x)\cdot\left(1-\frac{1}{2}\cdot x\right)\right],[x],[0.1],1000,1000\right)[1]$

$$[0.4753049232] \quad (8)$$

>

> #4a)

> $Orb\left(HW3(u,v,w),[u,v,w],\left[\frac{1}{3},\frac{1}{3},\frac{1}{3}\right],1,1\right)[1][2]$

$$\frac{1}{2} \quad (9)$$

> #4b)

> $Orb\left(HW3(u,v,w),[u,v,w],\left[\frac{1}{3},\frac{1}{3},\frac{1}{3}\right],999,999\right)[1][2]$

$$\frac{1}{2} \quad (10)$$

>

> #5a)

> $Orb\left(HW3g(u,v,w,[[.1,.2,.1],[.1,.1,.1],[.1,.1,.1]]),[u,v,w],\left[\frac{1}{3},\frac{1}{3},\frac{1}{3}\right],1,1\right)[1][2]$

$$0.5000000001 \quad (11)$$

> $Orb\Big(HW3g(u,v,w,[[.1,.2,.1],[.1,.1,.1],[.1,.1,.1]]),[u,v,w],\left[\frac{1}{3},\frac{1}{3},\frac{1}{3}\right],999,$

$999\Big)[1][2]$

$$0.3974661806 \quad \textbf{(12)}$$

>

> *#6)*

> $OrbF\left(\left[\frac{1+x+y}{2+x+3\cdot y}, \frac{1+x+3\cdot y}{3+x+2\cdot y}\right], [x, y], [100, 1000], 1000, 1001\right)[1][2]$

$$0.7478789080 \quad \textbf{(13)}$$

> $OrbF\left(\left[\frac{1+x+y}{2+x+3\cdot y}, \frac{1+x+3\cdot y}{3+x+2\cdot y}\right], [x, y], [100, 1000], 1000, 1001\right)[2][2]$

$$0.7478789080 \quad \textbf{(14)}$$

>

> *#7a)*

> $SEquP(SIRS(s, i, 0.05, 0.5, 100, 1000), [s, i])$

$$\{[1000., 0.]\} \quad \textbf{(15)}$$

> $Dis(SIRS(s, i, 0.05, 0.5, 100, 1000), [s, i], [300, 300], 0.01, 20)[-1]$

$$[20.01, [999.9666301, 6.997984915 \times 10^{-686}]] \quad \textbf{(16)}$$

> *#7b)*

> $SEquP(SIRS(s, i, 1.4, 0.5, 100, 1000), [s, i])$

$$\{[71.42857143, 4.619758351]\} \quad \textbf{(17)}$$

> *#7c)*

> $SIRS(s, i, b, 0.5, 100, 1000)$

$$[-b\, s\, i + 500.0 - 0.5\, s - 0.5\, i, b\, s\, i - 100\, i] \quad \textbf{(18)}$$

> $SysEq := \{-b\, s\, i + 500.0 - 0.5\, s - 0.5\, i = 0, b\, s\, i - 100\, i = 0\}$

$$SysEq := \{-b\, s\, i + 500.0 - 0.5\, s - 0.5\, i = 0, b\, s\, i - 100\, i = 0\} \quad \textbf{(19)}$$

> $solve(SysEq, \{s, i\})$

$$\{i = 0., s = 1000.\}, \left\{i = \frac{0.4975124378\ (10.\,b - 1.)}{b}, s = \frac{100.}{b}\right\} \quad \textbf{(20)}$$

> $solve\left(0 = \frac{0.4975124378\ (10.\,b - 1.)}{b}, b\right)$

$$0.1000000000 \quad \textbf{(21)}$$

> $solve\left(1000 = \frac{100.}{b}, b\right)$

$$0.1000000000 \quad \textbf{(22)}$$

>

> *#8a)*

> $SEquP(GeneNet(0, 1, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])$

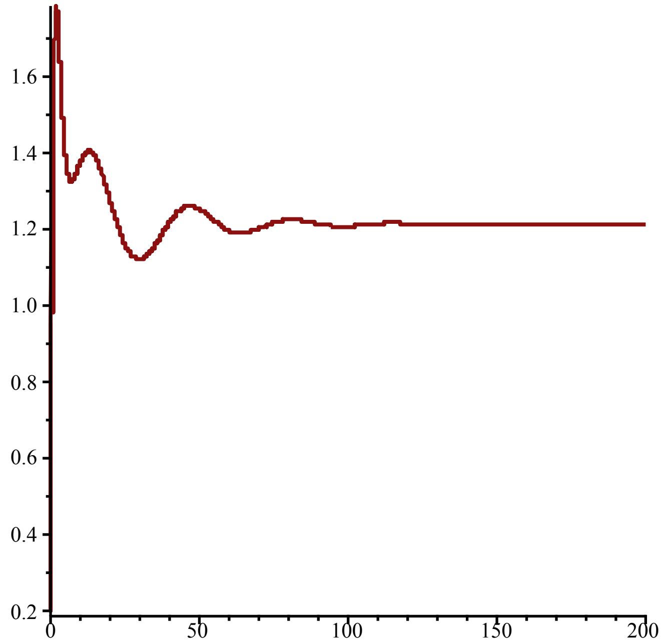
$$\{[0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038]\} \quad \textbf{(23)}$$

> *#8b)*

>

> $TimeSeries(GeneNet(0, 3, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3], [0.2, 0.1,$

0.3, 0.1, 0.4, 0.5], 0.01, 200, 1)



> *SEquP*(*GeneNet*(0, 3, 0.2, 2, *m1*, *m2*, *m3*, *p1*, *p2*, *p3*), [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*])

{ [1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663] } **(24)**

> *#8c)*

```
> for a from 3 by 1 to 50
  do
  print(a);
  SEquP(GeneNet(0, a, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3]) ;
  end do;
```

3

{ [1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663] }

4

{ [1.378796700, 1.378796700, 1.378796700, 1.378796700, 1.378796700, 1.378796700] }

5

{ [1.515980228, 1.515980228, 1.515980228, 1.515980228, 1.515980228, 1.515980228] }

6

{ [1.634365293, 1.634365293, 1.634365293, 1.634365293, 1.634365293, 1.634365293] }

7

{ [1.739203861, 1.739203861, 1.739203861, 1.739203861, 1.739203861, 1.739203861] }

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$$50$$
$$\varnothing \qquad (25)$$

```
> for a from 7 by 0.1 to 8
  do
  print(a);
```

```
SEquP(GeneNet(0, a, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3]) ;
end do;
```

7

{[1.739203861, 1.739203861, 1.739203861, 1.739203861, 1.739203861, 1.739203861]}

7.1

{[1.749079318, 1.749079318, 1.749079318, 1.749079318, 1.749079318, 1.749079318]}

7.2

{[1.758855227, 1.758855227, 1.758855227, 1.758855227, 1.758855227, 1.758855227]}

7.3

{[1.768534008, 1.768534008, 1.768534008, 1.768534008, 1.768534008, 1.768534008]}

7.4

∅

7.5

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7.6

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7.7

∅

7.8

∅

7.9

∅

8.0

∅ **(26)**

```
> for a from 7.3 by 0.01 to 7.4
do
print(a);
SEquP(GeneNet(0, a, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3]) ;
end do;
```

7.3

{[1.768534008, 1.768534008, 1.768534008, 1.768534008, 1.768534008, 1.768534008]}

7.31

{[1.769496635, 1.769496635, 1.769496635, 1.769496635, 1.769496635, 1.769496635]}

7.32

{[1.770458315, 1.770458315, 1.770458315, 1.770458315, 1.770458315, 1.770458315]}

7.33

{[1.771419053, 1.771419053, 1.771419053, 1.771419053, 1.771419053, 1.771419053]}

7.34

{[1.772378849, 1.772378849, 1.772378849, 1.772378849, 1.772378849, 1.772378849]}

7.35

{[1.773337706, 1.773337706, 1.773337706, 1.773337706, 1.773337706, 1.773337706]}

7.36

{[1.774295627, 1.774295627, 1.774295627, 1.774295627, 1.774295627, 1.774295627]}

7.37

{[1.775252614, 1.775252614, 1.775252614, 1.775252614, 1.775252614, 1.775252614]}

7.38

{[1.776208668, 1.776208668, 1.776208668, 1.776208668, 1.776208668, 1.776208668]}

7.39

{[1.777163792, 1.777163792, 1.777163792, 1.777163792, 1.777163792, 1.777163792]}

7.40

∅ **(27)**

\>

\> *#9a,b)*

\> *SEquP*(*ChemoStat*(*N*, *C*, 2.5, 2.7), [*N*, *C*])

{[5.083333333, 0.6666666667]} **(28)**

\>

\> *#10)*

\> $P := \left[\left[0.2, \frac{0.8}{8}, \frac{0.8}{8}, \frac{0.8}{8}, \frac{0.8}{8}, \frac{0.8}{8}, \frac{0.8}{8}, \frac{0.8}{8}, \frac{0.8}{8}\right], \left[\frac{0.8}{8}, 0.2, \frac{0.8}{8}, \frac{0.8}{8}, \frac{0.8}{8}, \frac{0.8}{8}, \frac{0.8}{8}, \frac{0.8}{8}, \frac{0.8}{8}\right], \left[\frac{0.8}{8}, \frac{0.8}{8}, 0.2, \frac{0.8}{8}, \frac{0.8}{8}, \frac{0.8}{8}, \frac{0.8}{8}, \frac{0.8}{8}, \frac{0.8}{8}\right], \left[\frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, 0.4, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}\right], \left[\frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, 0.4, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}\right], \left[\frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, 0.4, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}\right], \left[\frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, 0.6, \frac{0.4}{8}, \frac{0.4}{8}\right], \left[\frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, 0.6, \frac{0.4}{8}\right], \left[\frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, 0.6\right]\right]$

P := [[0.2, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000], [0.1000000000, 0.2, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000], [0.1000000000, 0.1000000000, 0.2, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000], [0.07500000000, 0.07500000000, 0.07500000000, 0.4, 0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000], [0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000, 0.4, 0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000], [0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000, 0.4, 0.07500000000, 0.07500000000, 0.07500000000], [0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.6, 0.05000000000, 0.05000000000], **(29)** [0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.6, 0.05000000000], [0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.6]]

> $evalm(P^{1000})$

[[0.07692307655, 0.07692307655, 0.07692307655, 0.1025641021, 0.1025641021, 0.1025641021, 0.1538461530, 0.1538461530, 0.1538461530], **(30)**
[0.07692307655, 0.07692307655, 0.07692307655, 0.1025641021, 0.1025641021, 0.1025641021, 0.1538461530, 0.1538461530, 0.1538461530],
[0.07692307655, 0.07692307655, 0.07692307655, 0.1025641021, 0.1025641021, 0.1025641021, 0.1538461530, 0.1538461530, 0.1538461530],
[0.07692307651, 0.07692307651, 0.07692307651, 0.1025641020, 0.1025641020, 0.1025641020, 0.1538461530, 0.1538461530, 0.1538461530],
[0.07692307651, 0.07692307651, 0.07692307651, 0.1025641020, 0.1025641020, 0.1025641020, 0.1538461530, 0.1538461530, 0.1538461530],
[0.07692307651, 0.07692307651, 0.07692307651, 0.1025641020, 0.1025641020, 0.1025641020, 0.1538461530, 0.1538461530, 0.1538461530],
[0.07692307651, 0.07692307651, 0.07692307651, 0.1025641020, 0.1025641020, 0.1025641020, 0.1538461530, 0.1538461530, 0.1538461530],
[0.07692307651, 0.07692307651, 0.07692307651, 0.1025641020, 0.1025641020, 0.1025641020, 0.1538461530, 0.1538461530, 0.1538461530],
[0.07692307651, 0.07692307651, 0.07692307651, 0.1025641020, 0.1025641020, 0.1025641020, 0.1538461530, 0.1538461530, 0.1538461530]]

>