```
> #Nikita John, Final Exam
> read "C:/Users/nikit/.maplesoft/DMB.txt" :
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

--- **(1)**

```
> #NOTE: These questions will be out of order, as I like to skip around when completing exams.
      The answers will be organized in NikitaJohnFinal.txt
> #1
  F := proc(n) option remember
  if n = 0 then
  1 :
  elif n = 1 then
  1 :
  elif n = 2 then
  2 :
```

```
else
expand(2·F(n − 1) - F(n − 3)) :
fi:
end:
```

```
> F999 := evalf(seq(F(i), i = 999));
  F1000 := evalf(seq(F(i), i = 1000));
  Ans1 := evalf( F1000 / F999 );
```

$$F999 := 4.346655769 \times 10^{208}$$

$$F1000 := 7.033036771 \times 10^{208}$$

$$Ans1 := 1.618033989 \quad (2)$$

```
> #2 (a) & (b)
  f := 5/2 ·x· (1 − x) · (1 − 1/2 ·x) :
  fEq := EquP([f], [x]);
  fSEq := SEquP([f], [x]);
```

$$fEq := \{[0], [1], [2]\}$$

$$fSEq := \{[1.]\} \quad (3)$$

```
> #2(c)
  fDiff := dsolve( {diff(x(t), t) = 5/2 ·x(t)·(1 − x(t))·(1 − 1/2 ·x(t)), x(0) = 0.1}, {x(t)} );
```

$$fDiff := x(t) = \frac{19\, e^{\frac{5t}{2}}}{81 \left(-\frac{1}{\sqrt{1 + \frac{19\, e^{\frac{5t}{2}}}{81}}} - 1 \right) \left(-\frac{19\, e^{\frac{5t}{2}}}{81} - 1 \right)} \quad (4)$$

```
> evalf(subs(t = 100, fDiff) );
```

$$x(100) = 0.9999999999 \quad (5)$$

```
> #3 (a) & (b)
  Digits := 10;
  evalf(FP([f], [x]));
  SFP([f], [x]);
```

$$Digits := 10$$

$$\{[0.], [0.475304923], [2.524695077]\}$$

$$\{[0.475304923]\} \quad (6)$$

```
> #3 (c)
  Orb([f], [x], [0.1], 1000, 1000)[1];
```

$$[0.4753049232] \quad (7)$$

```
> #6 - NOTE: I tried running this using Orb, but it didn't really work because n was so large. So, I
```

figured that since n is pretty much infinity, the value of y(n) would be the stable equilibrium value of y(n), which is given below:

$Digits := 10;$

$g := \left[\frac{(1+x+y)}{2+x+3\cdot y}, \frac{(1+x+3\cdot y)}{3+x+2\cdot y} \right]:$

$SFP(g, [x, y]);$

$$Digits := 10$$

$$\{[0.4705902280, 0.7478789082]\} \quad \textbf{(8)}$$

> *#10 (a) & (b)*

$with(LinearAlgebra):$

$A := Matrix([[0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1], [0.1, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1], [0.1, 0.1, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1], [0.075, 0.075, 0.075, 0.4, 0.075, 0.075, 0.075, 0.075, 0.075], [0.075, 0.075, 0.075, 0.075, 0.4, 0.075, 0.075, 0.075, 0.075], [0.075, 0.075, 0.075, 0.075, 0.075, 0.4, 0.075, 0.075, 0.075], [0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.6, 0.05, 0.05], [0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.6, 0.05], [0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.6]]);$

$$A := \begin{bmatrix} 0.2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.075 & 0.075 & 0.075 & 0.4 & 0.075 & 0.075 & 0.075 & 0.075 & 0.075 \\ 0.075 & 0.075 & 0.075 & 0.075 & 0.4 & 0.075 & 0.075 & 0.075 & 0.075 \\ 0.075 & 0.075 & 0.075 & 0.075 & 0.075 & 0.4 & 0.075 & 0.075 & 0.075 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.6 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.6 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.6 \end{bmatrix} \quad \textbf{(9)}$$

> $MatrixPower(A, 1000);$

[[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160], **(10)**

[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],

[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],

[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],

[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160,

0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107,
0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160,
0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107,
0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160,
0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107,
0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160,
0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107,
0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160,
0.153846153846160]]

```
> #9 (a) & (b)
  Help(ChemoStat);
  chemoEq := ChemoStat(N, C, 2.5, 2.7);
  Dis(chemoEq, [N, C], [1, 1], 0.01, 10)[1001];
```

ChemoStat(N,C,a1,a2): The Chemostat continuous-time dynamical system with N=Bacterial poplulation densitty, and C=nutient Concentration in growth chamber (see Table 4.1 of Edelstein-Keshet, p. 122)

with paramerts a1, a2, Equations (19a_, (19b) in Edelestein-Keshet p. 127 (section 4.5, where they are called alpha1, alpha2). a1 and a2 can be symbolic or numeric. Try:

ChemoStat(N,C,a1,a2);

ChemoStat(N,C,2,3);

$$chemoEq := \left[\frac{2.5\, C\, N}{C+1} - N, -\frac{C\, N}{C+1} - C + 2.7 \right]$$

$$[10.01, [5.083019282, 0.6667361650]] \qquad \textbf{(11)}$$

```
> #4 (a) & (b)
```

$$M := Matrix\left(\left[\left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]\right]\right):$$

$basicHW := HW3g(u, v, w, M);$

$$Orb\left(basicHW, [u, v, w], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], 2, 2\right)[1];$$

$$Orb\left(basicHW, [u, v, w], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], 1000, 1000\right)[1];$$

$$basicHW := \left[\frac{\frac{1}{3} u^2 + \frac{1}{3} u\, v + \frac{1}{12} v^2}{\frac{1}{3} u^2 + \frac{2}{3} u\, v + \frac{1}{3} v^2 + \frac{2}{3} u\, w + \frac{2}{3} v\, w + \frac{1}{3} w^2}, \right.$$

$$\left[\frac{\frac{1}{3}\,u\,v+\frac{2}{3}\,u\,w+\frac{1}{6}\,v^2+\frac{1}{3}\,v\,w}{\frac{1}{3}\,u^2+\frac{2}{3}\,u\,v+\frac{1}{3}\,v^2+\frac{2}{3}\,u\,w+\frac{2}{3}\,v\,w+\frac{1}{3}\,w^2},\ \frac{\frac{1}{12}\,v^2+\frac{1}{3}\,v\,w+\frac{1}{3}\,w^2}{\frac{1}{3}\,u^2+\frac{2}{3}\,u\,v+\frac{1}{3}\,v^2+\frac{2}{3}\,u\,w+\frac{2}{3}\,v\,w+\frac{1}{3}\,w^2}\right]$$

$$\left[\frac{1}{4},\frac{1}{2},\frac{1}{4}\right]$$

$$\left[\frac{1}{4},\frac{1}{2},\frac{1}{4}\right] \tag{12}$$

> #8 (a)

Help(GeneNet)

GeneNet(a0,a,b,n,m1,m2,m3,p1,p2,p3): The contiuous-time dynamical system, with quantities m1, m2,m3,p1,p2,p3, due to M. Elowitz and S. Leibler described in the Ellner-Guckenheimer book, Eq. (4.1) (chapter 4, p. 112) and parameers a0 (called alpha_0 there),a (called alpha there), b (called beta there) and n. Try: GeneNet(0,0.5,0.2,2,m1,m2,m3,p1,p2,p3); **(13)**

> protModel := GeneNet(0, 1, 0.2, 2, m1, m2, m3, p1, p2, p3);
SEquP(protModel, [m1, m2, m3, p1, p2, p3]);
Digits := 10;

$$protModel := \left[-m1+\frac{1}{p3^2+1},\ -m2+\frac{1}{p1^2+1},\ -m3+\frac{1}{p2^2+1},\ -0.2\,p1+0.2\,m1,\ -0.2\,p2+0.2\,m2,\ -0.2\,p3+0.2\,m3\right]$$

{[0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038]}

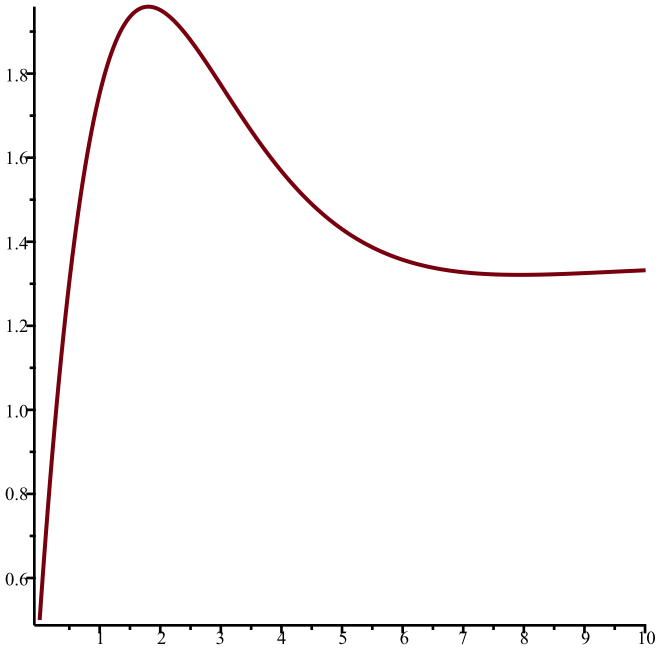
$$Digits := 10 \tag{14}$$

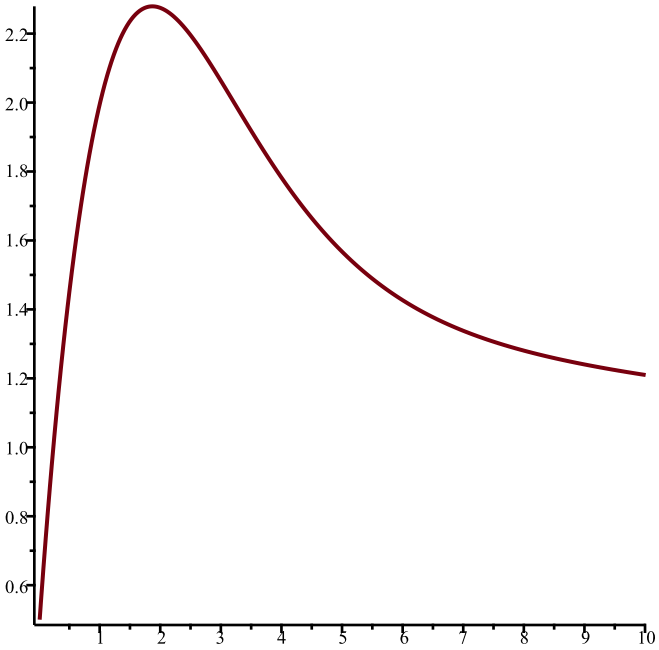
> #8 (b)

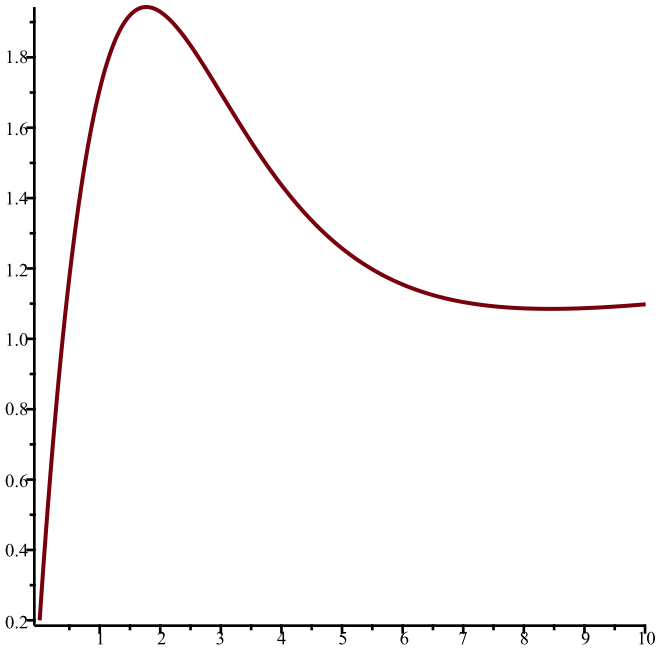
protModelModified := GeneNet(0, 3, 0.2, 2, m1, m2, m3, p1, p2, p3);
TimeSeries(protModelModified, [m1, m2, m3, p1, p2, p3], [0.5, 0.5, 0.2, 0.1, 0.3, 0.4], 0.01, 10, 1);
TimeSeries(protModelModified, [m1, m2, m3, p1, p2, p3], [0.5, 0.5, 0.2, 0.1, 0.3, 0.4], 0.01, 10, 2); TimeSeries(protModelModified, [m1, m2, m3, p1, p2, p3], [0.5, 0.5, 0.2, 0.1, 0.3, 0.4], 0.01, 10, 3); TimeSeries(protModelModified, [m1, m2, m3, p1, p2, p3], [0.5, 0.5, 0.2, 0.1, 0.3, 0.4], 0.01, 10, 4); TimeSeries(protModelModified, [m1, m2, m3, p1, p2, p3], [0.5, 0.5, 0.2, 0.1, 0.3, 0.4], 0.01, 10, 5); TimeSeries(protModelModified, [m1, m2, m3, p1, p2, p3], [0.5, 0.5, 0.2, 0.1, 0.3, 0.4], 0.01, 10, 6);
SEquP(protModelModified, [m1, m2, m3, p1, p2, p3]);

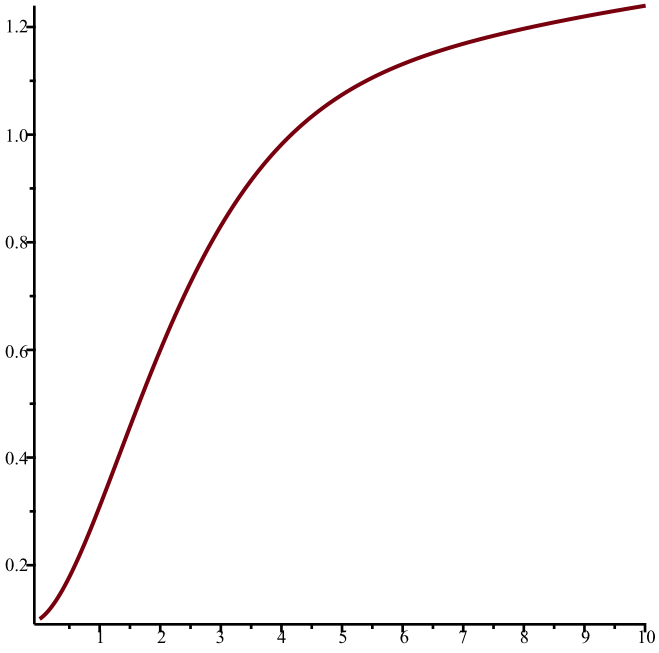
$$protModelModified := \left[-m1+\frac{3}{p3^2+1},\ -m2+\frac{3}{p1^2+1},\ -m3+\frac{3}{p2^2+1},\ -0.2\,p1\right.$$

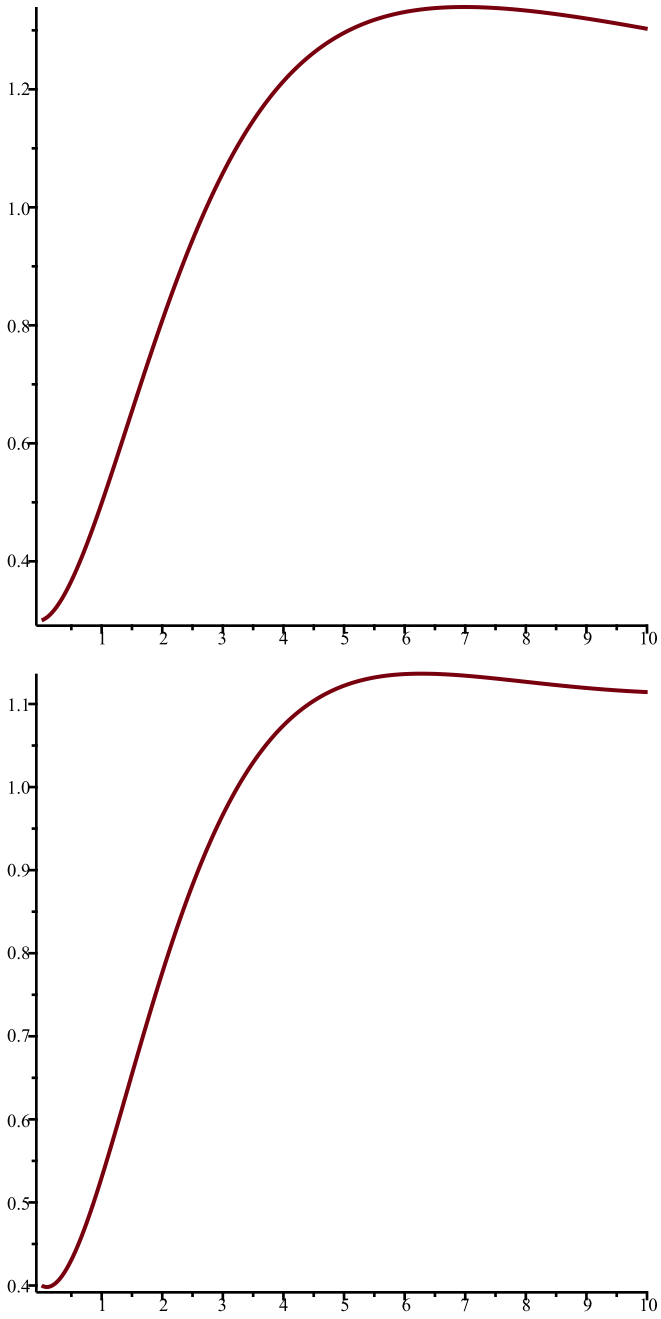
$$+ 0.2\,m1,\ -0.2\,p2 + 0.2\,m2,\ -0.2\,p3 + 0.2\,m3\Big]$$






1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
1
2
3
4
5
6
7
8
9
10


1.2
1.0
0.8
0.6
0.4
0.2
1
2
3
4
5
6
7
8
9
10



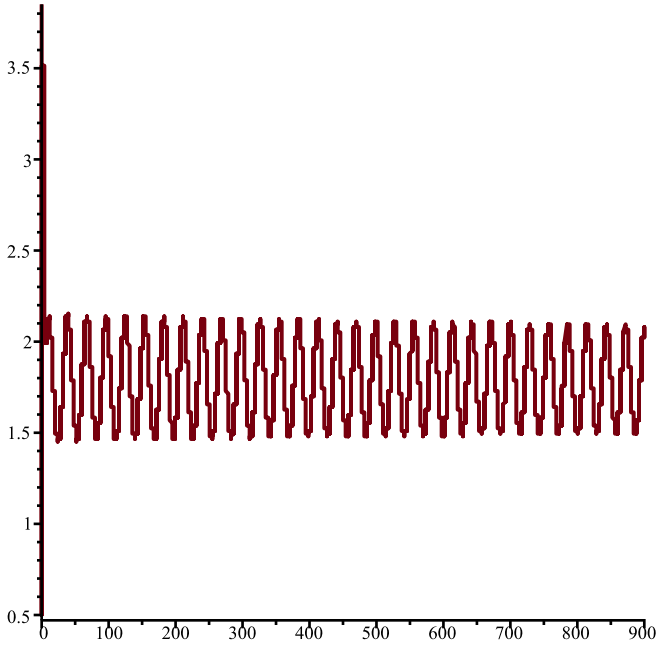
$$\{[1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663]\} \quad \textbf{(15)}$$

> *#8 (c) #I only did one graph because it took so long for the graphs to generate. It's hard to see in this graph, but the oscillations stay consistent, indicating that this model is unstable. This is further supported by SEquP returning an empty set for the set of stable equilibrium points.*

```
protModelSemi := GeneNet(0, 7.4, 0.2, 2, m1, m2, m3, p1, p2, p3);
TimeSeries(protModelSemi, [m1, m2, m3, p1, p2, p3], [0.5, 0.5, 0.2, 0.1, 0.3, 0.4], 0.01, 900,
    1);
SEquP(protModelSemi, [m1, m2, m3, p1, p2, p3]);
```

$$protModelSemi := \left[-m1 + \frac{7.4}{p3^2 + 1}, -m2 + \frac{7.4}{p1^2 + 1}, -m3 + \frac{7.4}{p2^2 + 1}, -0.2\,p1 + 0.2\,m1, \right.$$

$$-0.2\,p2 + 0.2\,m2,\ -0.2\,p3 + 0.2\,m3\Big]$$

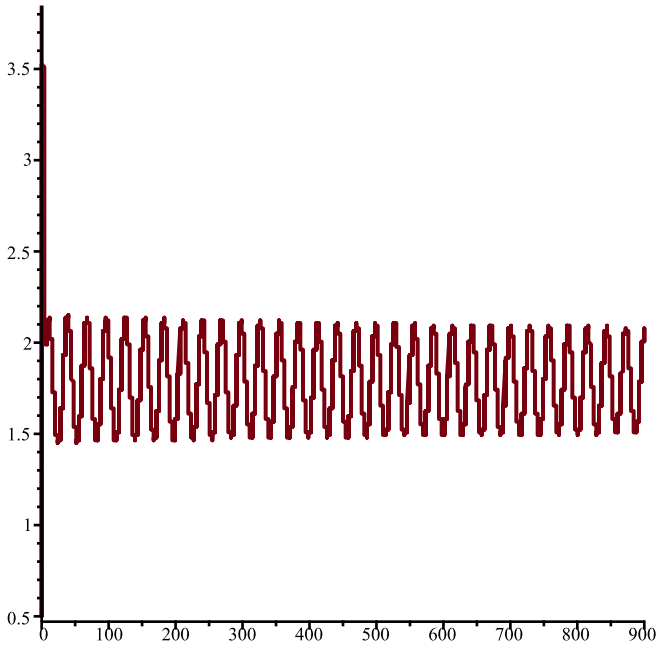


$\varnothing$ **(16)**

```
> protModelSemi := GeneNet(0, 7.39, 0.2, 2, m1, m2, m3, p1, p2, p3);
    #It's hard to see but you can slowly see the curves converge because the amplitude of the
    oscillations is getting smaller
  TimeSeries(protModelSemi, [m1, m2, m3, p1, p2, p3], [0.5, 0.5, 0.2, 0.1, 0.3, 0.4], 0.01, 900,
    1);
  SEquP(protModelSemi, [m1, m2, m3, p1, p2, p3]);
```

$$protModelSemi := \left[-m1 + \frac{7.39}{p3^2 + 1},\ -m2 + \frac{7.39}{p1^2 + 1},\ -m3 + \frac{7.39}{p2^2 + 1},\ -0.2\,p1 + 0.2\,m1,\right.$$

$$\left.-0.2\,p2 + 0.2\,m2,\ -0.2\,p3 + 0.2\,m3\right]$$



$$\{[1.777163792, 1.777163792, 1.777163792, 1.777163792, 1.777163792, 1.777163792]\} \quad \textbf{(17)}$$

> *#5 (a) & (b)*

$$MEdit := Matrix\left(\left[\left[\frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]\right]\right):$$

$$basicHW := HW3g(u, v, w, MEdit);$$

$$evalf\left(Orb\left(basicHW, [u, v, w], \left[\frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right], 2, 2\right)\right)[1];$$

$$OrbF\left(basicHW, [u, v, w], \left[\frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right], 1000, 1000\right)[1];$$

$$basicHW := \left[\frac{\frac{1}{3}u^2 + \frac{1}{2}uv + \frac{1}{12}v^2}{\frac{1}{3}u^2 + uv + \frac{1}{3}v^2 + \frac{2}{3}uw + \frac{2}{3}vw + \frac{1}{3}w^2}, \frac{\frac{1}{2}uv + \frac{2}{3}uw + \frac{1}{6}v^2 + \frac{1}{3}vw}{\frac{1}{3}u^2 + uv + \frac{1}{3}v^2 + \frac{2}{3}uw + \frac{2}{3}vw + \frac{1}{3}w^2}, \frac{\frac{1}{12}v^2 + \frac{1}{3}vw + \frac{1}{3}w^2}{\frac{1}{3}u^2 + uv + \frac{1}{3}v^2 + \frac{2}{3}uw + \frac{2}{3}vw + \frac{1}{3}w^2}\right]$$

$$[0.3055555556, 0.4986449864, 0.1957994580]$$

$$[0.5512669097, 0.3974661803, 0.05126690984] \quad \textbf{(18)}$$

> *#7 (a) & (b)*

#I believe something is wrong in the problem given, partially because of the results I get

when I just put beta in and partially because it doesn't match the format of other problems similar to this that we recieved in class. I emailed about it but it wasn't answered, so I did both versions to show what I got when adhering strictly to the problem versus using what we did in the past.

$beta1 := SIRS(s, i, 0.05, 0.5, 100, 1000);$

$Dis(beta1, [s, i], [300, 300], 0.01, 10)[1001];$

$$\beta 1 := [-0.05\, s\, i + 500.0 - 0.5\, s - 0.5\, i,\ 0.05\, s\, i - 100\, i]$$

$$[10.01, [994.9849412, 2.033393038 \times 10^{-384}]] \quad \textbf{(19)}$$

> $beta2 := SIRS(s, i, 1.4, 0.5, 100, 1000);$

$Dis(beta2, [s, i], [300, 300], 0.01, 10)[1001];$

$$\beta 2 := [-1.4\, s\, i + 500.0 - 0.5\, s - 0.5\, i,\ 1.4\, s\, i - 100\, i]$$

$$[10.01, [\text{Float}(\infty), \text{Float}(-\infty)]] \quad \textbf{(20)}$$

> *#In the following code above, it implies that there is 6 removed after a long time when beta = 0.05, and there is 0 removed when beta = 1.4. However, the floating point numbers do not have a real world significance, which makes me doubt the validity of these answers. Here is the following answers I get when following what we did in HW20 where beta = $x \cdot \frac{v}{N}$ where x is some real number*

> $beta1MyWay := SIRS\left(s, i, 0.05 \cdot \left(\frac{100}{1000}\right), 0.5, 100, 1000\right);$

$Dis(beta1MyWay, [s, i], [300, 300], 0.01, 10)[1001];$

$$beta1MyWay := [-0.005000000000\, s\, i + 500.0 - 0.5\, s - 0.5\, i,\ 0.005000000000\, s\, i - 100\, i]$$

$$[10.01, [995.3014874, 4.220120542 \times 10^{-1377}]] \quad \textbf{(21)}$$

> $beta2MyWay := SIRS\left(s, i, 1.4 \cdot \left(\frac{100}{1000}\right), 0.5, 100, 1000\right);$

$Dis(beta2MyWay, [s, i], [300, 300], 0.01, 10)[1001];$

$$beta2MyWay := [-0.1400000000\, s\, i + 500.0 - 0.5\, s - 0.5\, i,\ 0.1400000000\, s\, i - 100\, i]$$

$$[10.01, [735.3632819, 16.40557146]] \quad \textbf{(22)}$$

> *#I was especiallly convinced that the second way was better because it gave actual numbers for the second value, which is greater than the cutoff value. When beta is greater than the cutoff value, then the epidemic persists, and there should still be some sick individuals in the population, which was not shown when you follow the problem strictly. Hence my edits.*