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> **read**("C:/Users/mudas/Documents/DMB.txt")

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This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

> *#1*

> $g := Orbk(3, z, 2\,z[1] - z[3], [1, 1, 2], 1000, 1000)$

$g :=$ **(2)**

[
4346655768693745643568852767504062580256466051737178040248172908953655541794 9\
0518904038798400792551692959225930803226347752096896232398733224711616429964 4\
0906533187938298969649928516003704476137795166849228875]

> $h := Orbk(3, z, 2\,z[1] - z[3], [1, 1, 2], 999, 999)$

$h :=$ **(3)**

[
26863810024485359386146727202142923967616609318986952340123175997617981700247\
88168933836965448335656419182785616144335631297667364221035032463485041037768\
0367334151172899169723197082763985615764450078474174626]

> # Compute g divided by h to get 1.61803398875

>

#2

> $F := \left[\frac{5}{2} x \cdot (1 - x) \cdot \left(1 - \frac{1}{2} x\right)\right]$

$$F := \left[\frac{5\,x\,(1 - x)\left(1 - \frac{x}{2}\right)}{2}\right] \quad \textbf{(4)}$$

> #a)

> $EquP(F, [x])$

$$\{[0], [1], [2]\} \quad \textbf{(5)}$$

> #b)

> $SEquP(F, [x])$

$$\{[1.]\} \quad \textbf{(6)}$$

> #c)

> # Since 1 is the stable equilibrium point, it is expected that t = 100 would be 1.
Check:

> $x1 := dsolve\left(\left\{diff(x(t), t) = \frac{5}{2} x(t) \cdot (1 - x(t)) \cdot \left(1 - \frac{1}{2} x(t)\right), x(0) = 0.1\right\}, x(t)\right)$

$$x1 := x(t) = \frac{19\,e^{\frac{5\,t}{2}}}{81\left(-\frac{19\,e^{\frac{5\,t}{2}}}{81} - 1\right)\left(-\frac{1}{\sqrt{1 + \frac{19\,e^{\frac{5\,t}{2}}}{81}}} - 1\right)} \quad \textbf{(7)}$$

> $eval(x1, t = 100)$

$$x(100) = \frac{19\,e^{250}}{81\left(-\frac{19\,e^{250}}{81} - 1\right)\left(-\frac{1}{\sqrt{1 + \frac{19\,e^{250}}{81}}} - 1\right)} \quad \textbf{(8)}$$

> # This evaluates to 1.

>

> #3)

> $F1 := \left[\frac{5}{2}x\cdot(1-x)\cdot\left(1-\frac{1}{2}x\right)\right]$

$$F1 := \left[\frac{5\,x\,(1-x)\left(1-\frac{x}{2}\right)}{2}\right] \quad \textbf{(9)}$$

> #a)

> $FP(F1, [x])$

$$\left\{[0], \left[\frac{3}{2}-\frac{\sqrt{105}}{10}\right], \left[\frac{3}{2}+\frac{\sqrt{105}}{10}\right]\right\} \quad \textbf{(10)}$$

> #b)

> $SFP(F1, [x])$

$$\{[0.475304923]\} \quad \textbf{(11)}$$

> #c)

> $Orb(F1, [x], [0.1], 1000, 1000)$

$$[[0.4753049232]] \quad \textbf{(12)}$$

>

> #4

> #a)

> $Orb\left(\left(HW3(u, v, w), [u, v, w], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], 1, 1\right)\right)$

$$\left[\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right]\right] \quad \textbf{(13)}$$

> *# v represents Aa. The proportion of the second generation that would have Aa is 0.5.*

> #b)

> $Orb\left(\left(HW3(u, v, w), [u, v, w], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], 999, 999\right)\right)$

$$\left[\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right]\right] \quad \textbf{(14)}$$

> *# v represents Aa. The proportion of the second generation that would have Aa is 0.5.*

>

> #5

> $A := Matrix([[.1, .2, .1], [.1, .1 , .1], [.1, .1, .1]])$

$$A := \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} \quad \textbf{(15)}$$

> *# Change the matrix position M[1][2] to 0.2 to represent AA females twice as likely to mate with Aa male*

> #a)

> $Orb\left(HW3g(u, v, w, A), [u, v, w], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], 1, 1\right)$

$$[[0.2750000001, 0.5000000001, 0.2250000000]] \quad \textbf{(16)}$$

> # v represents Aa. The proportion of the second generation that would have Aa is 0.5.

> #b)

> $Orb\left(HW3g(u, v, w, A), [u, v, w], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], 999, 999\right)$

$$[[0.5512669096, 0.3974661806, 0.05126690988]] \quad (17)$$

> # v represents Aa. The proportion of the second generation that would have Aa is 0.3974661806.

>

> #6

> $F2 := \left[\frac{(1+x+y)}{2+x+3y}, \frac{(1+x+3y)}{3+x+2y}\right]$

$$F2 := \left[\frac{1+x+y}{2+x+3y}, \frac{1+x+3y}{3+x+2y}\right] \quad (18)$$

> $SFP(F2, [x, y])$

$$\{[0.4705902280, 0.7478789082]\} \quad (19)$$

> # By definition, the function will always end up at approximately y = 0.7478789082, since it is a stable fixed point. Therefore, y(y (1000000000000000000000000000000000000000000000000000000) will be 0.7478789082.

>

> #7

> #a)

> $F := SIRS(s, i, 0.05, 0.5, 100, 1000)$

$$F := [-0.05\, s\, i + 500.0 - 0.5\, s - 0.5\, i, 0.05\, s\, i - 100\, i] \quad (20)$$

> $SEquP(F, [s, i])$

$$\{[1000., 0.]\} \quad (21)$$

> # To get removed individuals, compute N - s - i.

> # The number of removed individuals at β = 0.05 is 0. (1000 - 1000 - 0) = 0

> #b)

> $F := SIRS(s, i, 1.4, 0.5, 100, 1000)$

$$F := [-1.4\, s\, i + 500.0 - 0.5\, s - 0.5\, i, 1.4\, s\, i - 100\, i] \quad (22)$$

> $SEquP(F, [s, i])$

$$\{[71.42857143, 4.619758351]\} \quad (23)$$

> # The number of removed individuals at β = 1.4 is approximately 923 (1000 - 71.42857143 - 4.619758351) = 923.951670219

> #c)

> $F := SIRS(s, i, 0.1, 0.5, 100, 1000)$

$$F := [-0.1\, s\, i + 500.0 - 0.5\, s - 0.5\, i, 0.1\, s\, i - 100\, i] \quad (24)$$

> $SEquP(F, [s, i])$

$$\varnothing \quad (25)$$

> # The cutoff appears to be β = 0.1 due to no stable fixed points.
> # To check, evaluate β = 0.099999 and β = 0.100001 :

> $F := SIRS(s, i, 0.099999, 0.5, 100, 1000)$

$$F := [-0.099999\, s\, i + 500.0 - 0.5\, s - 0.5\, i,\ 0.099999\, s\, i - 100\, i] \quad \textbf{(26)}$$

> $SEquP(F, [s, i])$

$$\{[1000., 0.]\} \quad \textbf{(27)}$$

> $F := SIRS(s, i, 0.100001, 0.5, 100, 1000)$

$$F := [-0.100001\, s\, i + 500.0 - 0.5\, s - 0.5\, i,\ 0.100001\, s\, i - 100\, i] \quad \textbf{(28)}$$

> $SEquP(F, [s, i])$

$$\{[999.9900001, 0.00004975074627]\} \quad \textbf{(29)}$$

> *# The number of infected individuals is still 0 close at 0.1 (shown by β = 0.099999), and changes to non-zero once past 0.1 (shown by β =0.100001). However, there are no stable fixed points at 0.1 itself. Therefore, β = 0.1 is the cutoff for non-zero number of infected people.*

>

> *#8*

> *#a)*

> $F := GeneNet(0, 1, 0.2, 2, m1, m2, m3, p1, p2, p3)$

$$F := \left[-m1 + \frac{1}{p3^2 + 1},\ -m2 + \frac{1}{p1^2 + 1},\ -m3 + \frac{1}{p2^2 + 1},\ -0.2\, p1 + 0.2\, m1,\ -0.2\, p2 + 0.2\, m2,\ -0.2\, p3 + 0.2\, m3\right] \quad \textbf{(30)}$$

> $SEquP(F, [m1, m2, m3, p1, p2, p3])$

$$[0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038] \quad \textbf{(31)}$$

> *# The horizonatal asymptote is at 0.6823278038. By definition, the height of the asymptote is the value of the stable equilibrium point.*

> *#b)*

> $F := GeneNet(0, 3, 0.2, 2, m1, m2, m3, p1, p2, p3)$

$$F := \left[-m1 + \frac{3}{p3^2 + 1},\ -m2 + \frac{3}{p1^2 + 1},\ -m3 + \frac{3}{p2^2 + 1},\ -0.2\, p1 + 0.2\, m1,\ -0.2\, p2 + 0.2\, m2,\ -0.2\, p3 + 0.2\, m3\right] \quad \textbf{(32)}$$

> $SEquP(F, [m1, m2, m3, p1, p2, p3])$

$$\{[1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663]\} \quad \textbf{(33)}$$

> *# The horizonatal asymptote is at 1.213411663. By definition, the height of the asymptote is the value of the stable equilibrium point.*

> *#c)*

> $F := GeneNet(0, 7.393, 0.2, 2, m1, m2, m3, p1, p2, p3)$

$$F := \left[-m1 + \frac{7.393}{p3^2 + 1},\ -m2 + \frac{7.393}{p1^2 + 1},\ -m3 + \frac{7.393}{p2^2 + 1},\ -0.2\, p1 + 0.2\, m1,\ -0.2\, p2 + 0.2\, m2,\ -0.2\, p3 + 0.2\, m3\right] \quad \textbf{(34)}$$

> $SEquP(F, [m1, m2, m3, p1, p2, p3])$

(35)

$$\{[1.777450148, 1.777450148, 1.777450148, 1.777450148, 1.777450148, 1.777450148]\} \quad (35)$$

> *# There are no stable equilibrium points at a = 7.394. However, at a = 7.393, there is stable equilibrium. Therefore, there is no longer a stable equilibrium starting a = 7.394 .*

> *# Check a = 7.393:*

> $F := GeneNet(0, 7.393, 0.2, 2, m1, m2, m3, p1, p2, p3)$

$$F := \left[-m1 + \frac{7.393}{p3^2 + 1}, -m2 + \frac{7.393}{p1^2 + 1}, -m3 + \frac{7.393}{p2^2 + 1}, -0.2\,p1 + 0.2\,m1, -0.2\,p2 + 0.2\,m2, -0.2\,p3 + 0.2\,m3 \right] \quad (36)$$

> $SEquP(F, [m1, m2, m3, p1, p2, p3])$

$$\{[1.777450148, 1.777450148, 1.777450148, 1.777450148, 1.777450148, 1.777450148]\} \quad (37)$$

> *# This shows that there is stable equilibrium at a = 7.393, but not at a = 7.394.*

>

> *#9*

> $F := ChemoStat(N, C, 2.5, 2.7)$

$$F := \left[\frac{2.5\,C\,N}{C + 1} - N, -\frac{C\,N}{C + 1} - C + 2.7 \right] \quad (38)$$

> $SEquP(F, [N, C])$

$$\{[5.083333333, 0.6666666667]\} \quad (39)$$

> *#a*

> *# The value of bacterial population density after a long time will be 5.08333333, since it is a stable fixed point.*

> *#b*

> *# The value of nutrient concentration after a long time will be 0.66666667, since it is a stable fixed point.*

>

> *#10*

> $A := Matrix\left(\left[[0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1], [0.1, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1], [0.1, 0.1, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1], \left[\frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, 0.4, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}\right], \left[\frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, 0.4, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}\right], \left[\frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}, 0.4, \frac{0.6}{8}, \frac{0.6}{8}, \frac{0.6}{8}\right], \left[\frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, 0.6, \frac{0.4}{8}, \frac{0.4}{8}\right], \left[\frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, 0.6, \frac{0.4}{8}\right], \left[\frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, \frac{0.4}{8}, 0.6\right]\right]\right)$

$$A := [[0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1], \quad (40)$$
$$[0.1, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1],$$
$$[0.1, 0.1, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1],$$

[0.07500000000, 0.07500000000, 0.07500000000, 0.4, 0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000],
[0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000, 0.4, 0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000],
[0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000, 0.4, 0.07500000000, 0.07500000000, 0.07500000000],
[0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.6, 0.05000000000, 0.05000000000],
[0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.6, 0.05000000000],
[0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.6]]

> A^{1000}

[[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160], **(41)**
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107,

0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160]]

> *#a*

> *# The probability of a surfer being on web-page 1 is 0.0769230769230801, or approximately 7.69 %*

> *#b*

> *# The probability of a surfer being on web-page 9 is 0.153846153846160, or approximately 15.385 %*

>