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> read "/Users/maxmekhanikov/Downloads/DMB.txt" :
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

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> # Max Mekhanikov – Final Exam
> # Q1
> Help(Orbk)
```

Orbk(k,z,f,INI,K1,K2): Given a positive integer k, a letter (symbol), z, an expression f of z[1], ..., z[k] (representing a multi-variable function of the variables z[1],...,z[k]
a vector INI representing the initial values [x[1],..., x[k]], and (in applications) positive integres K1 and K2, outputs the
values of the sequence starting at n=K1 and ending at n=K2. of the sequence satisfying the difference equation

x[n]=f(x[n-1],x[n-2],..., x[n-k+1]):

This is a generalization to higher-order difference equation of procedure Orb(f,x,x0,K1,K2). For example, try:

*Orbk(1,z,5/2*z[1]*(1-z[1]),[0.5],1000,1010);*

To get the Fibonacci sequence, type:

Orbk(2,z,z[1]+z[2],[1,1],1000,1010);

To get the part of the orbit between n=1000 and n=1010, of the 3rd order recurrence given in Eq. (4) of the Ladas-Amleh paper

https://sites.math.rutgers.edu/~zeilberg/Bio21/AmlehLadas.pdf

with initial conditions x(0)=1, x(1)=3, x(2)=5, Type:

Orbk(3,z,z[2]/(z[2]+z[3]),[1.,3.,5.],1000,1010);

To get the part of the orbit between n=1000 and n=1010, of the 3rd order recurrence given in Eq. (5) of the Ladas-Amleh paper

with initial conditions x(0)=1, x(1)=3, x(2)=5, Type:

Orbk(3,z,(z[1]+z[3])/z[2],[1.,3.,5.],1000,1010);

To get the part of the orbit between n=1000 and n=1010, of the 3rd order recurrence given in Eq. (6) of the Ladas-Amleh paper

with initial conditions x(0)=1, x(1)=3, x(2)=5, Type:

Orbk(3,z,(1+z[3])/z[1],[1.,3.,5.],1000,1010);

To get the part of the orbit between n=1000 and n=1010, of the 3rd order recurrence given in Eq. (7) of the Ladas-Amleh paper

with initial conditions x(0)=1, x(1)=3, x(2)=5, Type:

Orbk(3,z,(1+z[1])/(z[2]+z[3]),[1.,3.,5.],1000,1010); **(2)**

> $Q1 := Orbk(3, z, 2 \cdot z[1] - z[3], [1., 1., 2.], 999, 1000)$

$$Q1 := [2.686380976 \times 10^{208}, 4.346655726 \times 10^{208}] \quad (3)$$

> $\frac{Q1[2]}{Q1[1]}$

$$1.618033989 \quad (4)$$

> **# Q2**

> $Help(EquP)$

EquP(F,x): Given a transformation F in the list of variables finds all the Equilibrium points of the continuous-time dynamical system x'(t)=F(x(t))

*EquP([5/2*x*(1-x)],[x]);*

EquP([y(1-x-y),x*(3-2*x-y)],[x,y]);* **(5)**

> $EquP\left(\left[\left(\frac{5}{2}\cdot x\right)\cdot(1-x)\cdot\left(1-\frac{x}{2}\right)\right],[x]\right)$

$$\{[0],[1],[2]\} \quad (6)$$

> $Help(SEquP)$

SEquP(F,x): Given a transformation F in the list of variables finds all the Stable Equilibrium points of the continuous-time dynamical system x'(t)=F(x(t))

*SEquP([5/2*x*(1-x)],[x]]);*

SEquP([y(1-x-y),x*(3-2*x-y)],[x,y]]);* **(7)**

> $SEquP\left(\left[\left(\frac{5}{2}\cdot x\right)\cdot(1-x)\cdot\left(1-\frac{x}{2}\right)\right],[x]\right)$

$$\{[1.]\} \quad (8)$$

> $Orb\left(\left[\frac{5}{2}\cdot x\cdot(1-x)\cdot\left(1-\frac{x}{2}\right)\right],[x],[0.1],100,101\right)[1]$

$$[0.4753049232] \quad (9)$$

> **# Q3**

> $EquP\left(\left[\left(\frac{5}{2}\cdot x\right)\cdot(1-x)\cdot\left(1-\frac{x}{2}\right)-x\right],[x]\right)$

$$\left\{[0],\left[\frac{3}{2}-\frac{\sqrt{105}}{10}\right],\left[\frac{3}{2}+\frac{\sqrt{105}}{10}\right]\right\} \quad (10)$$

> $SEquP\left(\left[\left(\frac{5}{2}\cdot x\right)\cdot(1-x)\cdot\left(1-\frac{x}{2}\right)-x\right],[x]\right)$

$$\{[0.475304923]\} \quad (11)$$

> $evalf\left(\frac{3}{2}-\frac{\sqrt{105}}{10}\right)$

$$0.475304923 \quad (12)$$

> $Orb\left(\left[\frac{5}{2}\cdot x\cdot(1-x)\cdot\left(1-\frac{x}{2}\right)\right],[x],[0.1],1000,1001\right)[1]$

$$[0.4753049232] \quad (13)$$

> **# Q4**

> $Orb\left(HW3(u,v,w),[u,v,w],\left[\frac{1}{3},\frac{1}{3},\frac{1}{3}\right],1,2\right)[1]$

$$\left[\frac{1}{4},\frac{1}{2},\frac{1}{4}\right] \quad (14)$$

> $Orb\left(HW3(u,v,w),[u,v,w],\left[\frac{1}{3},\frac{1}{3},\frac{1}{3}\right],1000,1001\right)[1]$

$$\left[\frac{1}{4},\frac{1}{2},\frac{1}{4}\right] \quad (15)$$

> $HW3(u,v,w)$

$$\left[u^2+v\,u+\frac{1}{4}\,v^2,\,v\,u+2\,u\,w+\frac{1}{2}\,v^2+v\,w,\,\frac{1}{4}\,v^2+v\,w+w^2\right] \quad (16)$$

> # In both the 2nd and 1000th generations, 1/2 of the population has a genotype of Aa.

> # Q5

> $Mating\_Prob := Matrix([[0.1, 0.1, 0.1], [0.2, 0.1, 0.1], [0.1, 0.1, 0.1]])$

$$Mating\_Prob := \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} \quad (17)$$

> $Orb\left(HW3g(u, v, w, Mating\_Prob), [u, v, w], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], 1, 2\right)[1]$

$$[0.2750000001, 0.5000000001, 0.2250000000] \quad (18)$$

> $Orb\left(HW3g(u, v, w, Mating\_Prob), [u, v, w], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], 1000, 1001\right)[1]$

$$[0.5512669096, 0.3974661806, 0.05126690988] \quad (19)$$

> # In the second generation, the probability of genotype Aa is 0.5 while in generation 1000 it decreases to 0.3974661806.

> # Q6

> $Orb\left(\left[\frac{(1 + x + y)}{(2 + x + 3 \cdot y)}, \frac{(1 + x + 3 \cdot y)}{(3 + x + 2 \cdot y)}\right], [x, y], [100, 1000], 1, 8\right)$

$$\left[\left[\frac{367}{1034}, \frac{3101}{2103}\right], \left[\frac{6152737}{14740107}, \frac{193317}{210895}\right], \left[\frac{7255709600299}{16063350995902}, \frac{12954736130137}{16322452596748}\right], \right. \quad (20)$$

$$\left[\frac{981201224499781296565 75487}{2111844944851285917662 29461}, \frac{502456840867618189395425035}{660601889284311460989102444}\right],$$

$$\left[\frac{10347943608363812868869388404480480277038524561 2016349}{22072312405792929475074825679549659973773029474 7954667}, \frac{17422016537645322731941726139983069389700399590 1320439}{23185571809299781286671756351364956001514721844 9936850}\right],$$

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762295281941618976131908704034548713440284379110641726361729319798470046579 8\
40597768164006051770105676 42]]

> $evalf\left(\frac{3101}{2103}\right)$

1.474560152 **(21)**

> $evalf\left(\frac{193317}{210895}\right)$

$$0.9166504659 \quad (22)$$

> $evalf\left(\frac{12954736130137}{16322452596748}\right)$

$$0.7936758311 \quad (23)$$

> $evalf\left(\frac{50245684086761818939542503 5}{66060188928431146098910244 4}\right)$

$$0.7606046077 \quad (24)$$

> $evalf\left(\frac{174220165376453227319417261399830693897003995901320439}{231855718092997812866717563513649560015147218449936850}\right)$

$$0.7514162981 \quad (25)$$

> evalf(
1905314749165509566822398875250955671618260482432214811501409914503879439 5\
526655181302316362980839246795603 9/
254428892597133189025129222739170378398641781511554652834162595730308540972\
162183805674202853353178280975126)

$$0.7488594278 \quad (26)$$

> **# Here the values become too large to evaluate so we analyze the first 10 digits instead**

> $evalf\left(\frac{2285648244}{3055066471}\right)$

$$0.7481500863 \quad (27)$$

> $evalf\left(\frac{3291991065}{4401329392}\right)$

$$0.7479538048 \quad (28)$$

> $evalf\left(\frac{2276862175}{3044342102}\right)$

$$0.7478995785 \quad (29)$$

> **# Based on the first several terms of the orbit, it becomes clear that the y coordinate approaches 0.74. During each iteration, the value decrements by a lower value than the difference prior. For example, from y(1) to y(2), the value changed by 1.474560152 − 0.9166504659 = 0.5579096861. This is a drastic difference when compared to y(8) to y(9) which changes only by 0.7479538048 − 0.7478995785 = 0.0000542263. Due to this decreasing step size with each iteration, I believe y would equal roughly 0.7465000000 in its long term behavior. Unfortunately, Maple is not able to calculate the orbit to that length because of how many digits are required, as shown above. If this was not the case, we could use Orb setting K1 = 1000 and K2 = 1010 for example, and get a better approximation.**

> **# Q7**

> *Help*(*SIRS*)

SIRS(s,i,beta,gamma,nu,N): The SIRS dynamical model with parameters beta,gamma, nu,N (see section 6.6 of Edelstein-Keshet), s is the number of

Susceptibles, i is the number of infected, (the number of removed is given by N-s-i). N is the total population. Try:

SIRS(s,i,beta,gamma,nu,N); **(30)**

> *SIRS*(300, 300, 0.05, 0.5, 100, 1000)

$[-4300.00, -25500.00]$ **(31)**

> *SIRS*(300, 300, 1.4, 0.5, 100, 1000)

$[-125800.0, 96000.0]$ **(32)**

> $SIRS\left(300, 300, \frac{1}{10}, 0.5, 100, 1000\right)$

$[-8800.0, -21000]$ **(33)**

> *SIRS*(*s*, *i*, beta, gamma, nu, *N*)

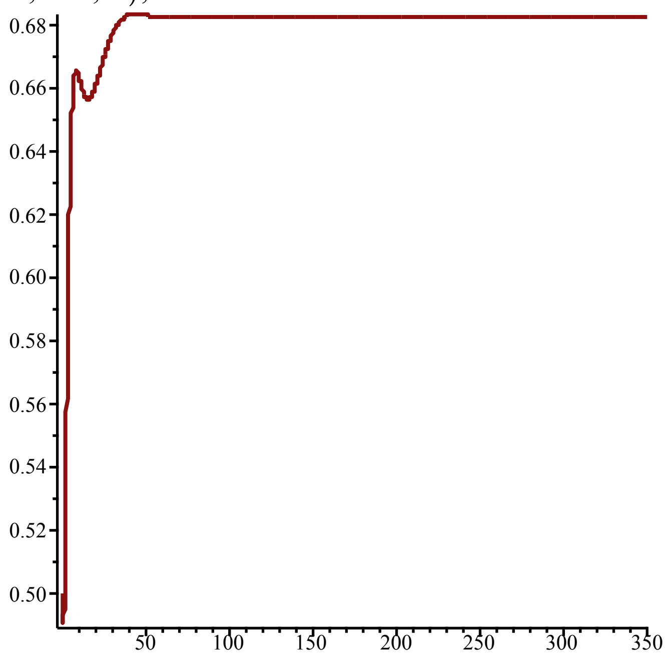
$[-\beta\, s\, i + \gamma\,(N - s - i), \beta\, s\, i - \nu\, i]$ **(34)**

> **# Q8**

> *SEquP*(*GeneNet*(0, 1, 0.2, 2, *m1*, *m2*, *m3*, *p1*, *p2*, *p3*), [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*])

{[0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038]} **(35)**

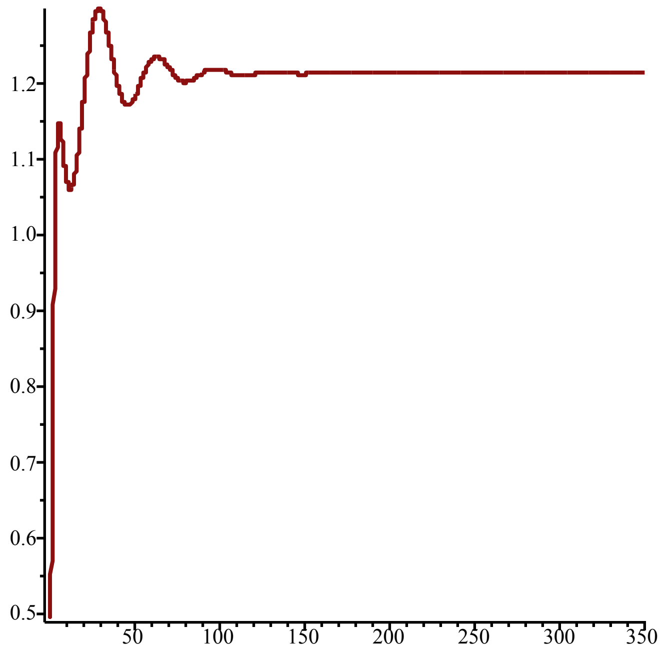
> *TimeSeries*(*GeneNet*(0, 1, 0.2, 2, *m1*, *m2*, *m3*, *p1*, *p2*, *p3*), [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [0.2, 0.1, 0.3, 0.1, 0.4, 0.5], .1, 350, 6);



> *SEquP*(*GeneNet*(0, 3, 0.2, 2, *m1*, *m2*, *m3*, *p1*, *p2*, *p3*), [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*])

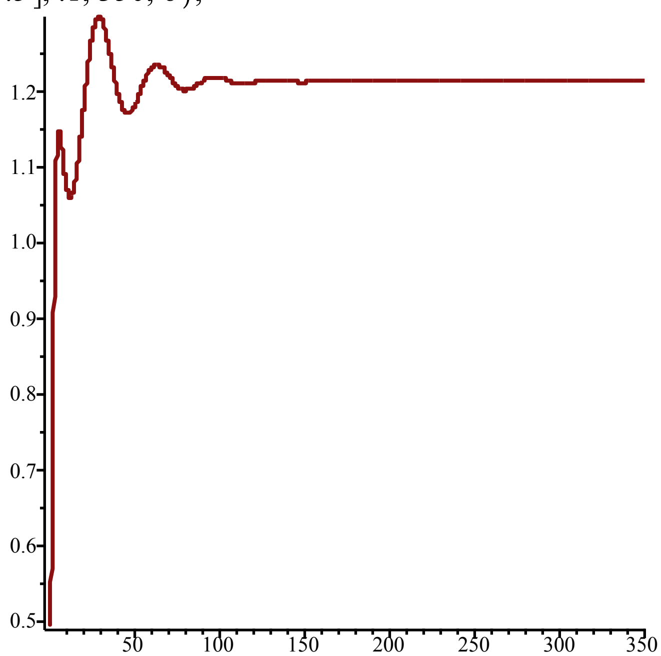
{[1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663]} **(36)**

> *TimeSeries*(*GeneNet*(0, 3, 0.2, 2, *m1*, *m2*, *m3*, *p1*, *p2*, *p3*), [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [0.2, 0.1, 0.3, 0.1, 0.4, 0.5], .1, 350, 6);

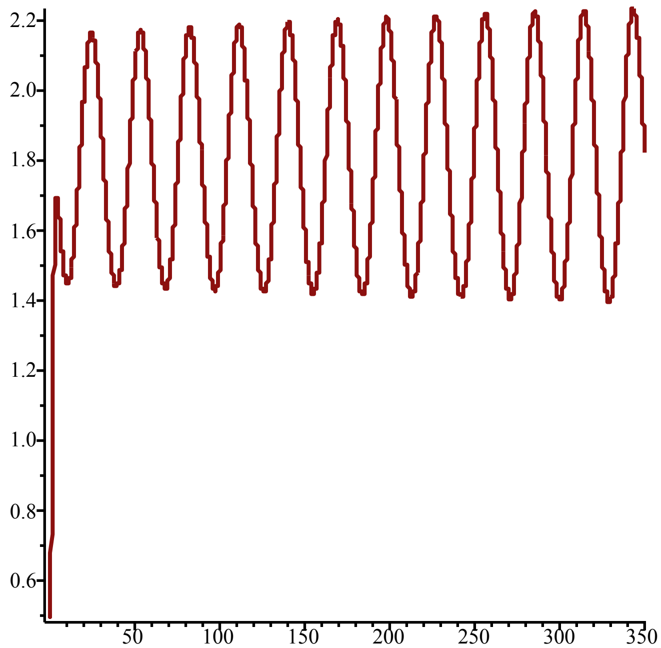


> # The horizontal axis still exists, however now at a height of 1.213411663 now when a = 3.

> *TimeSeries*(*GeneNet*(0, 3, 0.2, 2, *m1*, *m2*, *m3*, *p1*, *p2*, *p3*), [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [0.2, 0.1, 0.3, 0.1, 0.4, 0.5], .1, 350, 6);



> *TimeSeries*(*GeneNet*(0, 7.3, 0.2, 2, *m1*, *m2*, *m3*, *p1*, *p2*, *p3*), [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [0.2, 0.1, 0.3, 0.1, 0.4, 0.5], .1, 350, 6);



> $SEquP(GeneNet(0, 7.3, 0.2, 2, m1, m2, m3, p1, p2, p3),\ [m1, m2, m3, p1, p2, p3])$

$$\{[1.768534008, 1.768534008, 1.768534008, 1.768534008, 1.768534008, 1.768534008]\} \quad \textbf{(37)}$$

> $SEquP(GeneNet(0, 7.4, 0.2, 2, m1, m2, m3, p1, p2, p3),\ [m1, m2, m3, p1, p2, p3])$

$$\varnothing \quad \textbf{(38)}$$

```
> # At the value of a = 7.3, there is still a stable equilibrium as
  shown by the SEquP function. However, at a = 7.4, only 0.1 greater,
  the system changes behavior and a stable equilibrium no longer
  exists.
```

```
> # Q9
```

> $Help(ChemoStat)$

ChemoStat(N,C,a1,a2): The Chemostat continuous-time dynamical system with N=Bacterial poplulation densitty, and C=nutient Concentration in growth chamber (see Table 4.1 of Edelstein-Keshet, p. 122)

with paramerts a1, a2, Equations (19a_, (19b) in Edelestein-Keshet p. 127 (section 4.5, where they are called alpha1, alpha2). a1 and a2 can be symbolic or numeric. Try:

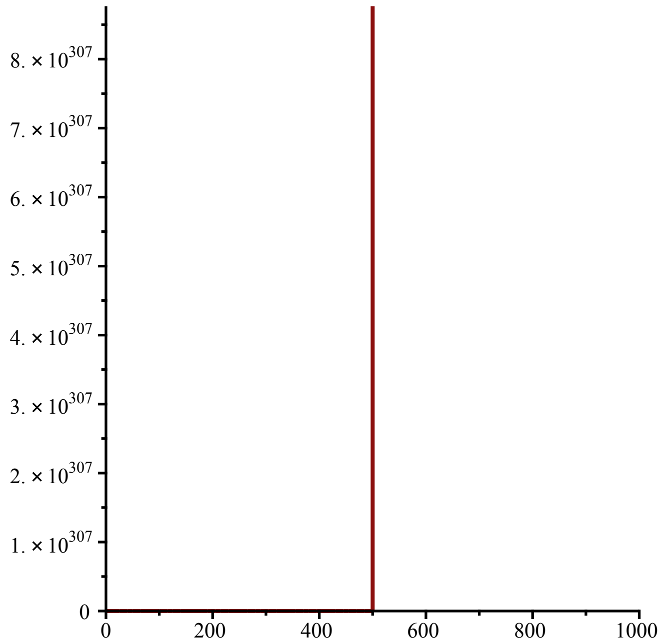
ChemoStat(N,C,a1,a2);

ChemoStat(N,C,2,3); **(39)**

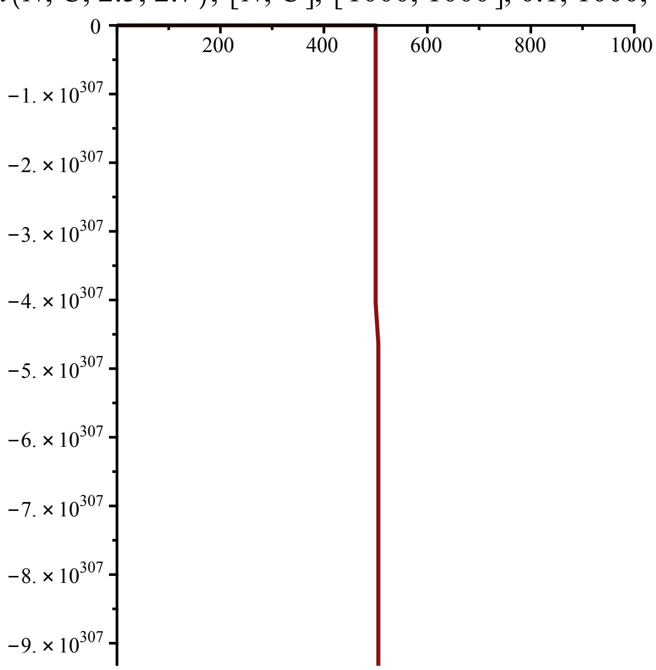
> $ChemoStat(N, C, a1, a2);$

$$\left[\frac{a1\, C\, N}{C+1} - N,\ -\frac{C\, N}{C+1} - C + a2\right] \quad \textbf{(40)}$$

> $TimeSeries(ChemoStat(N, C, 2.5, 2.7), [N, C], [1000, 1000], 0.1, 1000, 1)$



> $TimeSeries(ChemoStat(N, C, 2.5, 2.7), [N, C], [1000, 1000], 0.1, 1000, 2)$




```
> # After a very long time, the population of bacteria explodes to a
  very large number and the bacteria consume all of the nutrients,
  causing it to run out at the same time the bacteria population
  explodes.
>
  # Q10
```

> $M := Matrix\left(\left[\left[0.2, 0.1, 0.1, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.1}{2}, \frac{0.1}{2}, \frac{0.1}{2}\right], \left[0.1, 0.2, 0.1, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.1}{2}, \frac{0.1}{2}, \frac{0.1}{2}\right], \left[0.1, 0.1, 0.2, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.1}{2}, \frac{0.1}{2}, \frac{0.1}{2}\right], \left[0.1, 0.1, 0.1, \right.\right.\right.$

> $M := Matrix\left(\left[\left[0.2, 0.1, 0.1, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.1}{2}, \frac{0.1}{2}, \frac{0.1}{2}\right], \left[0.1, 0.2, 0.1, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.1}{2}, \frac{0.1}{2}, \frac{0.1}{2}\right], \left[0.1, 0.1, 0.2, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.1}{2}, \frac{0.1}{2}, \frac{0.1}{2}\right], \left[0.1, 0.1, 0.1, 0.4, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.1}{2}, \frac{0.1}{2}, \frac{0.1}{2}\right], \left[0.1, 0.1, 0.1, \frac{0.3}{4}, 0.4, \frac{0.3}{4}, \frac{0.1}{2}, \frac{0.1}{2}, \frac{0.1}{2}\right], \left[0.1, 0.1, 0.1, \frac{0.3}{4}, \frac{0.3}{4}, 0.4, \frac{0.1}{2}, \frac{0.1}{2}, \frac{0.1}{2}\right], \left[0.1, 0.1, 0.1, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.3}{4}, 0.6, \frac{0.1}{2}, \frac{0.1}{2}\right], \left[0.1, 0.1, 0.1, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.1}{2}, 0.6, \frac{0.1}{2}\right], \left[0.1, 0.1, 0.1, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.3}{4}, \frac{0.1}{2}, \frac{0.1}{2}, 0.6\right]\right]\right)$

$$M := \begin{bmatrix} 0.2 & 0.1 & 0.1 & 0.07500000000 & 0.07500000000 & 0.07500000000 & 0.05000000000 & 0.05000000000 & 0.05000000000 \\ 0.1 & 0.2 & 0.1 & 0.07500000000 & 0.07500000000 & 0.07500000000 & 0.05000000000 & 0.05000000000 & 0.05000000000 \\ 0.1 & 0.1 & 0.2 & 0.07500000000 & 0.07500000000 & 0.07500000000 & 0.05000000000 & 0.05000000000 & 0.05000000000 \\ 0.1 & 0.1 & 0.1 & 0.4 & 0.07500000000 & 0.07500000000 & 0.05000000000 & 0.05000000000 & 0.05000000000 \\ 0.1 & 0.1 & 0.1 & 0.07500000000 & 0.4 & 0.07500000000 & 0.05000000000 & 0.05000000000 & 0.05000000000 \\ 0.1 & 0.1 & 0.1 & 0.07500000000 & 0.07500000000 & 0.4 & 0.05000000000 & 0.05000000000 & 0.05000000000 \\ 0.1 & 0.1 & 0.1 & 0.07500000000 & 0.07500000000 & 0.07500000000 & 0.6 & 0.05000000000 & 0.05000000000 \\ 0.1 & 0.1 & 0.1 & 0.07500000000 & 0.07500000000 & 0.07500000000 & 0.05000000000 & 0.6 & 0.05000000000 \\ 0.1 & 0.1 & 0.1 & 0.07500000000 & 0.07500000000 & 0.07500000000 & 0.05000000000 & 0.05000000000 & 0.6 \end{bmatrix} \quad (41)$$

> $with(LinearAlgebra):$

> M^{1000}

$$\begin{bmatrix} 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 \\ 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 \\ 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 & 0.0769230769230791 \\ 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 \\ 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 \\ 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 & 0.102564102564106 \\ 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 \\ 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 \\ 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 & 0.153846153846158 \end{bmatrix} \quad (42)$$

(43)

> **# In the long run, there is a 0.0769230769230791 probability that the user will be at web-page 1 and a 0.153846153846158 probability of being at web-page 9.**

Max Mekhanikov – Final

1. $x(0) = 1$, $x(1) = 1$, $x(2) = 2$

$$x(n) = 2x(n-1) - x(n-3)$$

$$\frac{x(1000)}{x(999)} = 1.618$$

2.

a. $$x'(t) = \frac{5}{2}x(t) * (1 - x(t)) * \left(1 - \frac{x(t)}{2}\right)$$

$$F(x) = \frac{5}{2}x * (1-x) * \left(1 - \frac{x}{2}\right) = 0$$

$x = 0$, $x = 1$, $x = 2$ are equilibrium solutions.

b.

$$F'(x) = -\frac{5\left(1-\frac{x}{2}\right)x}{2} - \frac{5(1-x)x}{4} + \frac{5(1-x)\left(1-\frac{x}{2}\right)}{2}$$

$$F'(x) = \frac{15x^2 - 30x + 10}{4}$$

$$F'(0) = \frac{15(0) - 30(0) + 10}{4} = \frac{10}{4}$$

$F'(x) < 0$ not met $\rightarrow$ 0 is unstable

$$F'(1) = \frac{15 - 30 + 10}{4} = \frac{-5}{4} < 0$$

1 is a stable eq. solution

$$F'(2) = \frac{15(4) - 30(2) + 10}{4} = \frac{10}{4}$$

2 is unstable as well, $F'(2) > 0$

c.

(In Maple)

$x(0) = 0.1$, $x(100) = 0.4753$

3. a.

$$x(n) = \frac{5}{2} x(n-1) * (1 - x(n-1)) * \left(1 - \frac{x(n-1)}{2}\right)$$

$$F(x) = \frac{5}{2} x * (1-x) * \left(1 - \frac{x}{2}\right) = x$$

$$5x(1-x)(1 - x/2) = 2x$$

$$\frac{\frac{5x^3}{2} - \frac{15x^2}{2} + 3x = 0}{x}$$

$$2.5x^2 - 7.5x + 3 = 0$$

$$x = \frac{3}{2} \pm \frac{\sqrt{105}}{10}, \quad x = 0$$

confirmed w/ Maple ↗

Equil. solutions: $x = 0$, $x = \frac{3}{2} + \frac{\sqrt{105}}{10}$, $x = \frac{3}{2} - \frac{\sqrt{105}}{10}$

b. Stable equil. solutions: $|F'(x)| < 1$

$$F'(0) = \frac{15(0) - 30(0) + 10}{4} = \frac{10}{4} > 1$$

↳ not stable

$F'\left(3/2 + \frac{\sqrt{105}}{10}\right) \approx 7.468 > 1$ → not stable

$F'\left(3/2 - \frac{\sqrt{105}}{10}\right) \approx -0.218$, $|-0.218| < 1$

↳ $x = \frac{3}{2} - \frac{\sqrt{105}}{10}$ is a stable equil. solution c.

$x(0) = 0.01$, $x(1000) = 0.4753049232$

↳ w/ Maple

4. $HW3(u, v, w) =$

$\left[u^2 + vu + \frac{1}{4}v^2, vu + 2uw + \frac{1}{2}v^2, \frac{1}{4}v^2 + vw + w^2\right]$

$u = 1/3$, $v = 1/3$, $w = 1/3$

$[1/4, 1/2, 1/4]$

↓ AA ↓ Aa ↓ aa

Both 2^{nd} and 1000^{th} gen. have $1/2$ population w/ Aa genotype.

5.

Female

Male	AA	Aa	aa
AA	0.1	0.1	0.1
Aa	0.2	0.1	0.1
aa	0.1	0.1	0.1

7.

a.

$$\bar{S} = \frac{\nu}{\beta} = \frac{100}{0.05} = 2000$$

$$\bar{I} = \gamma \frac{N - \bar{S}}{\nu + \gamma} = 0.5 \frac{1000 - 2000}{100 + 0.5}$$

$$= -4.975$$

$$\bar{R} = \frac{\nu \bar{I}}{\gamma} = \frac{100(-4.975)}{0.5} = -995$$

↳ @ $\beta = 0.05$, 995 will be removed in the long run

b.

$$\bar{S} = \frac{100}{1.4} = 71.43$$

$$\bar{I} = 0.5 \frac{1000 - 71.43}{100 + 1.4} = 4.578$$

$$\bar{R} = \frac{100(4.578)}{0.5} = 916$$

→ @ $\beta = 1.4$, 916 will be removed in the long run c.

$$\frac{N\beta}{V} > 1$$

$$\frac{1000\,\beta}{100} > 1$$

$$10\beta > 1$$

$\beta > 1/10$ there will be a non-zero amount of infected people.