2. $\frac{5}{2}x(t)(1-x(t))(1-\frac{1}{2}x(t))$

a. $0 = 5/2\,x(1-x)(1-\frac{1}{2}x)$

$x = 0 \quad x = 1 \quad x = 2$

b. $f(x) = 5/2\,x(1-x)(1-\frac{1}{2}x)$

$= (5/2\,x - \frac{5}{2}x^2)(1-\frac{1}{2}x)$

$= 5/2\,x - \frac{5}{4}x^2 - \frac{5}{2}x^2 + \frac{5}{4}x^3$

$f(x) = 5/2\,x - \frac{15}{4}x^2 + \frac{5}{4}x^3$

$f'(x) = \frac{5}{2} - \frac{15}{8}x + \frac{5}{12}x^2$

$f'(0) = \frac{5}{2}$, unstable

$f'(1) = 25/24$, unstable

$f'(2) = 5/12$, unstable

c. Maple

3. $x_n = \frac{5}{2} x_{n-1}(1 - x_{n-1})(1 - \frac{1}{2} x_{n-1})$

a. $x = \frac{5}{2}x - \frac{15}{4}x^2 + \frac{5}{4}x^3$

$0 = \frac{3}{2}x - \frac{15}{4}x^2 + \frac{5}{4}x^3$

$0 = 6x - 15x^2 + \frac{5}{2}x^3$ $\quad x = 0$

$x = \frac{15 \pm \sqrt{225 - 4(6)(5)}}{10} = \frac{15 \pm \sqrt{105}}{10}$ 答

b. $f'(x) = \frac{5}{2} - \frac{15}{8}x + \frac{5}{12}x^2$

$|f'(0)| = \frac{5}{2}$, unstable

$\left|f'\left(\frac{15 + \sqrt{105}}{10}\right)\right| = 0.422$, stable, less than 1

$\left|f'\left(\frac{15 - \sqrt{105}}{10}\right)\right| = 0.475$, stable, less than 1

c. Maple

```
> read "/Users/jjj104/Documents/DMB.txt";
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

```
> read "/Users/jjj104/Documents/M5.txt";
> #1
  x := RecToSeq([1, 1, 2], [2, 0, -1], 999)[999];
  y := RecToSeq([1, 1, 2], [2, 0, -1], 1000)[1000];
  evalf(y/x);
```

$$x :=$$
$$268638100244853593861467272021429239676166093189869523401231759976179817\backslash$$
$$00247881689338369654483356564191827856161443356312976673642210350324634850\backslash$$
$$41037768036733415117289916972319708276398561576445007847417462\backslash$$
$$6$$

$$y :=$$
$$434665576869374564356885276750406258025646605173717804024817290895365554\backslash$$
$$17$$

```
94905189040387984007925516929592259308032263477520968962323987332247116164\
29964409065331879382989696499285160037044761377951668492288 75
                    1.618033989                                              (2)
```

```
> #2c
fDiff := dsolve( {diff(x(t), t) = 5/2 ·x(t)·(1 − x(t))·(1 − 1/2 ·x(t)), x(0) = 0.1}, {x(t)} );
```

$$fDiff := x(t) = \frac{19\,e^{\frac{5t}{2}}}{81\left(-\frac{19\,e^{\frac{5t}{2}}}{81}-1\right)\left(-\frac{1}{\sqrt{1+\frac{19\,e^{\frac{5t}{2}}}{81}}}-1\right)} \qquad (3)$$

```
> #3c
Orb( [ (5/2 ·x)·(1 − x)·(1 − 1/2 ·x) ], [x], [0.1], 1000, 1 );
                    [[0.4753049232]]                                         (4)
```

```
> #4
Orb( HW3(u, v, w), [u, v, w], [1/3, 1/3, 1/3], 2, 2 );
Orb( HW3(u, v, w), [u, v, w], [1/3, 1/3, 1/3], 1000, 1000 );
                    [[1/4, 1/2, 1/4]]
                    [[1/4, 1/2, 1/4]]                                        (5)
```

```
> #5
M := [[1/6, 2/3, 1/6], [1/3, 1/3, 1/3], [1/3, 1/3, 1/3]]:
OrbF( HW3g(u, v, w, M), [u, v, w], [1/3, 1/3, 1/3], 2, 2 );
OrbF( HW3g(u, v, w, M), [u, v, w], [1/3, 1/3, 1/3], 1000, 1000 );
          [[0.2647058824, 0.5000000000, 0.2352941177]]
          [[0.3757562480, 0.4935706703, 0.1306730817]]                       (6)
```

```
> #6
OrbF( [ (1 + x + y)/(2 + x + 3·y), (1 + x + 3·y)/(3 + x + 2·y) ], [x, y], [100, 1000], 1000, 1010 );

[[0.4705902280, 0.7478789080], [0.4705902280, 0.7478789080], [0.4705902280,   (7)
  0.7478789080], [0.4705902280, 0.7478789080], [0.4705902280, 0.7478789080],
  [0.4705902280, 0.7478789080], [0.4705902280, 0.7478789080], [0.4705902280,
```

0.7478789080], [0.4705902280, 0.7478789080], [0.4705902280, 0.7478789080],
[0.4705902280, 0.7478789080]]

> #7

$N := 1000;$
$v := 100;$
$g := 0.5;$
$ba := 0.05 \cdot \left(\frac{v}{N} \right);$
$bb := 1.4 \cdot \left(\frac{v}{N} \right);$
$afunc := SIRS(x, y, ba, g, v, N);$
$SEquP(afunc, [x, y]);$
$RemovedA := 1000 - (1000 + 0);$
$bfunc := SIRS(x, y, bb, g, v, N);$
$SEquP(bfunc, [x, y]);$
$RemovedB := 1000 - (700 + 1);$

$$N := 1000$$
$$v := 100$$
$$g := 0.5$$
$$ba := 0.005000000000$$
$$bb := 0.1400000000$$
$$afunc := [-0.005000000000\,x\,y + 500.0 - 0.5\,x - 0.5\,y,\ 0.005000000000\,x\,y - 100\,y]$$
$$\{[1000., 0.]\}$$
$$RemovedA := 0$$
$$bfunc := [-0.1400000000\,x\,y + 500.0 - 0.5\,x - 0.5\,y,\ 0.1400000000\,x\,y - 100\,y]$$
$$\{[714.2857143, 1.421464108]\}$$
$$RemovedB := 299 \qquad \textbf{(8)}$$

> #8a

$a0 := 0;$
$a := 1;$
$b := 0.2;$
$n := 2;$
$G := GeneNet(a0, a, b, n, m01, m02, m03, p01, p02, p03);$
$m1 := \mathrm{trunc}(evalf(rand() * 10^{(-11)}));$
$m2 := \mathrm{trunc}(evalf(rand() * 10^{(-11)}));$
$m3 := \mathrm{trunc}(evalf(rand() * 10^{(-11)}));$
$p1 := \mathrm{trunc}(evalf(rand() * 10^{(-11)}));$
$p2 := \mathrm{trunc}(evalf(rand() * 10^{(-11)}));$
$p3 := \mathrm{trunc}(evalf(rand() * 10^{(-11)}));$
$SEquP(G, [m01, m02, m03, p01, p02, p03]);$
$TimeSeries(G, [m01, m02, m03, p01, p02, p03], [m1, m2, m3, p1, p2, p3], 0.01, 10, 1);$

$$a0 := 0$$

$$a := 1$$

$$b := 0.2$$

$$n := 2$$

$$G := \left[-m01 + \frac{1}{p03^2 + 1}, -m02 + \frac{1}{p01^2 + 1}, -m03 + \frac{1}{p02^2 + 1}, -0.2\,p01 + 0.2\,m01, \right.$$
$$\left. -0.2\,p02 + 0.2\,m02, -0.2\,p03 + 0.2\,m03 \right]$$

$$m1 := 8$$

$$m2 := 8$$

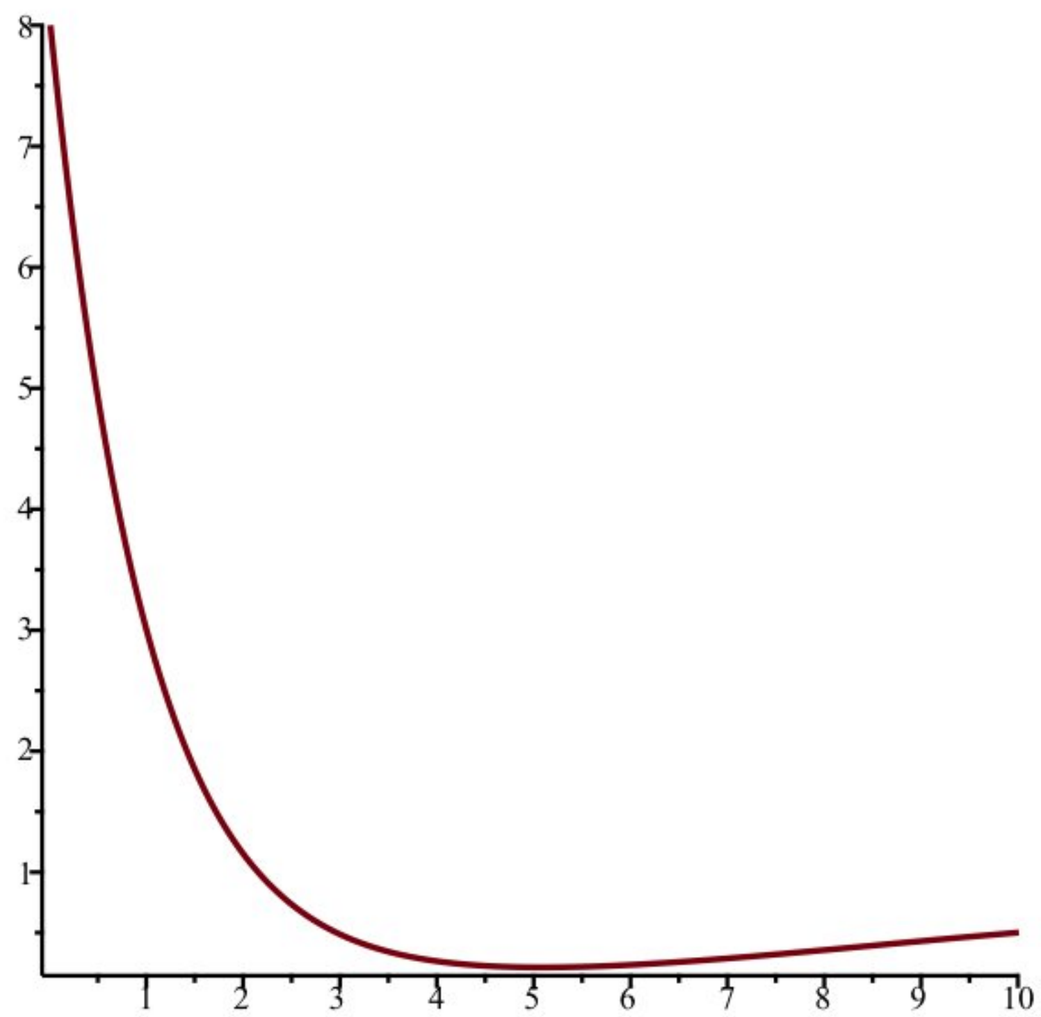
$$m3 := 9$$

$$p1 := 4$$

$$p2 := 5$$

$$p3 := 3$$

$$\{[0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038]\}$$




```
> #8b
  a0 := 0;
   a := 3;
   b := 0.2;
   n := 2;
  G := GeneNet(a0, a, b, n, m01, m02, m03, p01, p02, p03);
  m1 := trunc(evalf(rand( ) * 10^(-11)));
  m2 := trunc(evalf(rand( ) * 10^(-11)));
  m3 := trunc(evalf(rand( ) * 10^(-11)));
  p1 := trunc(evalf(rand( ) * 10^(-11)));
  p2 := trunc(evalf(rand( ) * 10^(-11)));
  p3 := trunc(evalf(rand( ) * 10^(-11)));
```

```
SEquP(G, [m01, m02, m03, p01, p02, p03]);
TimeSeries(G, [m01, m02, m03, p01, p02, p03], [m1, m2, m3, p1, p2, p3], 0.01, 10, 1);
```

$$a0 := 0$$

$$a := 3$$

$$b := 0.2$$

$$n := 2$$

$$G := \left[-m01 + \frac{3}{p03^2 + 1}, -m02 + \frac{3}{p01^2 + 1}, -m03 + \frac{3}{p02^2 + 1}, -0.2\,p01 + 0.2\,m01, -0.2\,p02 + 0.2\,m02, -0.2\,p03 + 0.2\,m03 \right]$$

$$m1 := 3$$

$$m2 := 6$$

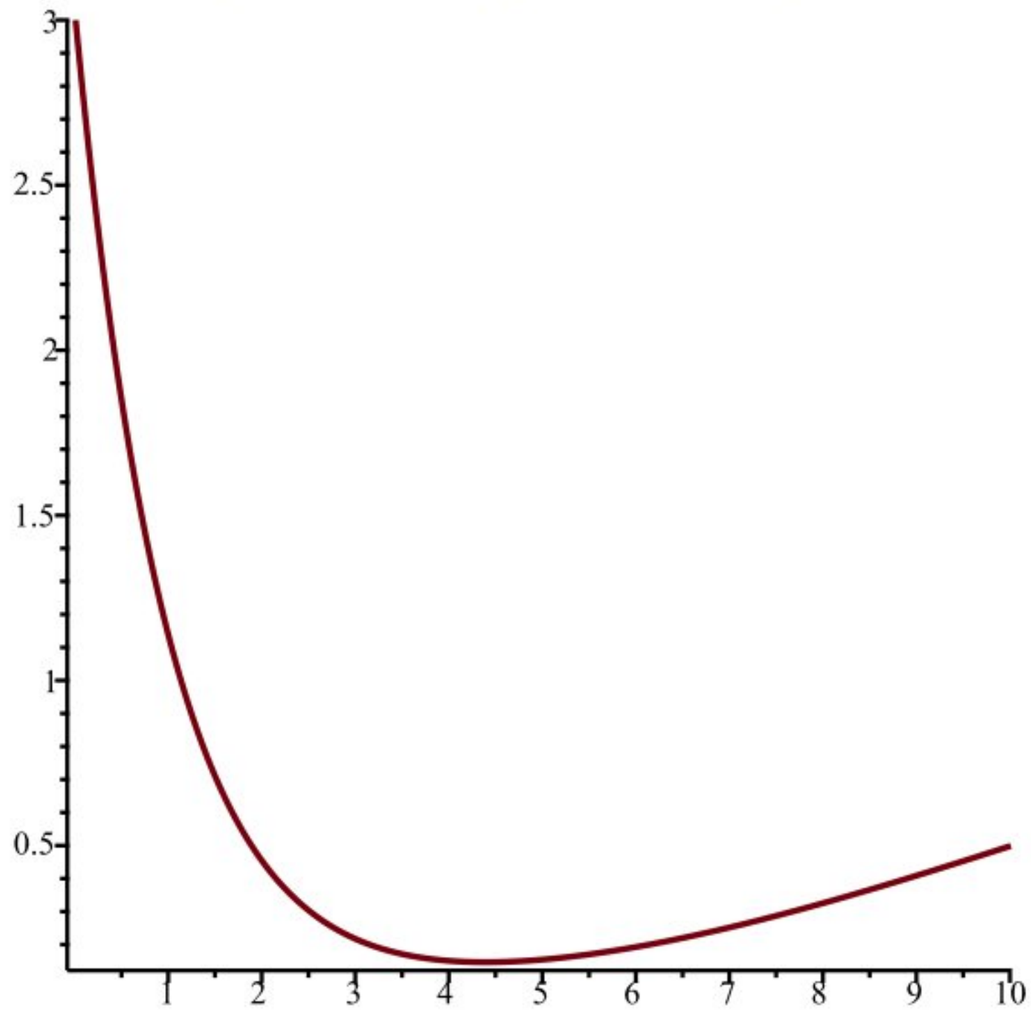
$$m3 := 9$$

$$p1 := 4$$

$$p2 := 3$$

$$p3 := 8$$

$$\{[1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663]\}$$




```
#8c
a0 := 0;
 a := 7.4;
 b := 0.2;
 n := 2;
G := GeneNet(a0, a, b, n, m01, m02, m03, p01, p02, p03);
m1 := trunc(evalf(rand( ) * 10^(-11)));
m2 := trunc(evalf(rand( ) * 10^(-11)));
m3 := trunc(evalf(rand( ) * 10^(-11)));
p1 := trunc(evalf(rand( ) * 10^(-11)));
p2 := trunc(evalf(rand( ) * 10^(-11)));
```

```
p3 := trunc(evalf(rand( ) * 10^( - 11) ) );
SEquP(G, [m01, m02, m03, p01, p02, p03 ]);
TimeSeries(G, [m01, m02, m03, p01, p02, p03 ], [m1, m2, m3, p1, p2, p3 ], 0.01, 10, 1 );
```

$$a0 := 0$$

$$a := 7.4$$

$$b := 0.2$$

$$n := 2$$

$$G := \left[-m01 + \frac{7.4}{p03^2 + 1}, -m02 + \frac{7.4}{p01^2 + 1}, -m03 + \frac{7.4}{p02^2 + 1}, -0.2\,p01 + 0.2\,m01, \right.$$

$$\left. -0.2\,p02 + 0.2\,m02, -0.2\,p03 + 0.2\,m03 \right]$$

$$m1 := 9$$

$$m2 := 6$$

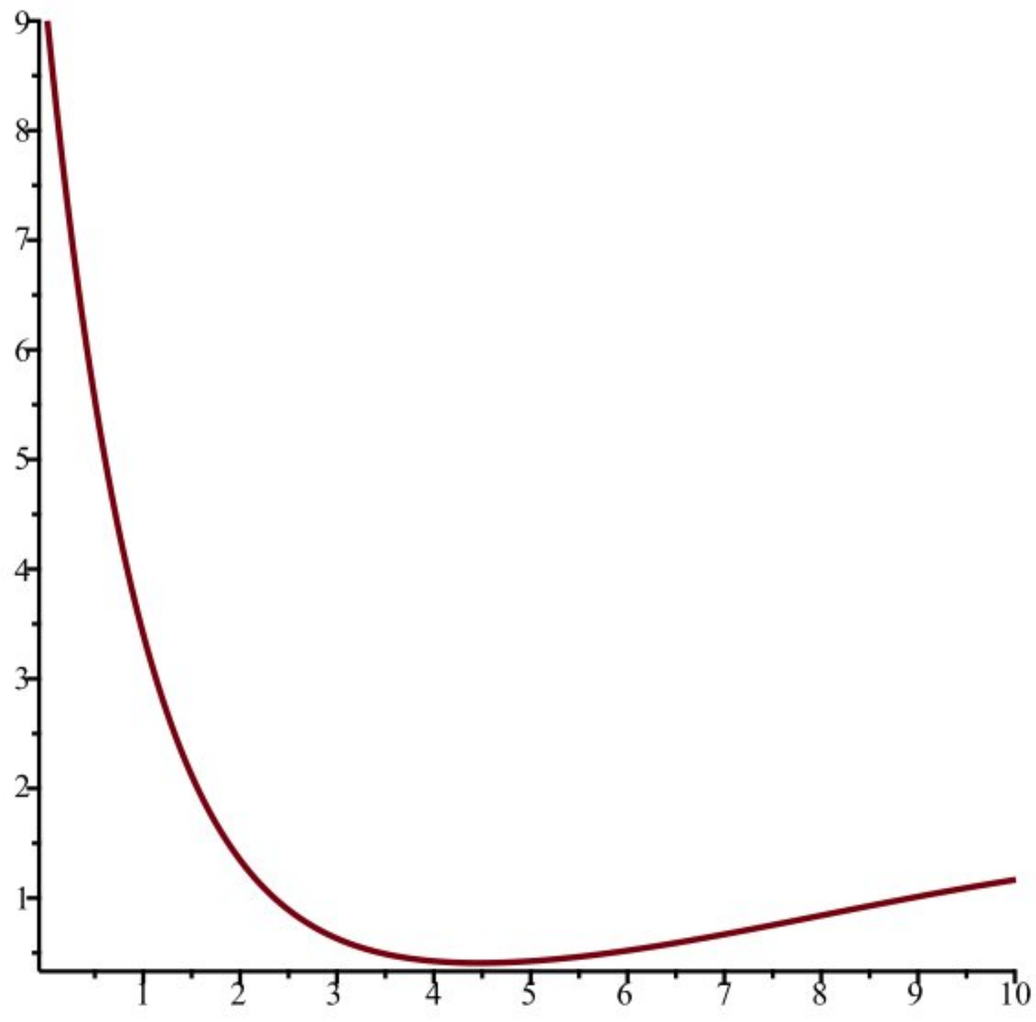
$$m3 := 7$$

$$p1 := 6$$

$$p2 := 6$$

$$p3 := 8$$

$$\varnothing$$




```
#9
chemoEq := ChemoStat(N, C, 2.5, 2.7);
Dis(chemoEq, [N, C], [1, 1 ], 0.01, 10)[1001 ];
```

$$chemoEq := \left[\frac{2.5\,C\,N}{C+1} - N, -\frac{C\,N}{C+1} - C + 2.7 \right]$$

$$[10.01, [5.083019282, 0.6667361650]] \qquad \textbf{(9)}$$

```
#10

M := Matrix( [ [0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1 ], [0.1, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1,
    0.1 ], [0.1, 0.1, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1 ], [0.075, 0.075, 0.075, 0.4, 0.075, 0.075,
```

0.075, 0.075, 0.075], [0.075, 0.075, 0.075, 0.075, 0.4, 0.075, 0.075, 0.075, 0.075], [0.075, 0.075, 0.075, 0.075, 0.075, 0.4, 0.075, 0.075, 0.075], [0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.6, 0.05, 0.05], [0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.6, 0.05], [0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.6]]);

$$evalf\left(M^{1000}\right);$$

$$M := \begin{bmatrix} 0.2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.075 & 0.075 & 0.075 & 0.4 & 0.075 & 0.075 & 0.075 & 0.075 & 0.075 \\ 0.075 & 0.075 & 0.075 & 0.075 & 0.4 & 0.075 & 0.075 & 0.075 & 0.075 \\ 0.075 & 0.075 & 0.075 & 0.075 & 0.075 & 0.4 & 0.075 & 0.075 & 0.075 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.6 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.6 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.6 \end{bmatrix}$$

(10)

[[0.076923076923076, 0.076923076923076, 0.076923076923076, 0.102564102564106, 0.102564102564106, 0.102564102564106, 0.153846153846159, 0.153846153846159, 0.153846153846159],
[0.076923076923076, 0.076923076923076, 0.076923076923076, 0.102564102564106, 0.102564102564106, 0.102564102564106, 0.153846153846159, 0.153846153846159, 0.153846153846159],
[0.076923076923076, 0.076923076923076, 0.076923076923076, 0.102564102564106, 0.102564102564106, 0.102564102564106, 0.153846153846159, 0.153846153846159, 0.153846153846159],
[0.076923076923095, 0.076923076923095, 0.076923076923095, 0.102564102564106, 0.102564102564106, 0.102564102564106, 0.153846153846159, 0.153846153846159, 0.153846153846159],
[0.076923076923095, 0.076923076923076, 0.076923076923076, 0.102564102564106, 0.102564102564106, 0.102564102564106, 0.153846153846159, 0.153846153846159, 0.153846153846159],
[0.076923076923095, 0.076923076923095, 0.076923076923095, 0.102564102564106, 0.102564102564106, 0.102564102564106, 0.153846153846159, 0.153846153846159, 0.153846153846159],
[0.076923076923076, 0.076923076923076, 0.076923076923076, 0.102564102564106, 0.102564102564106, 0.102564102564106, 0.153846153846159, 0.153846153846159, 0.153846153846159],
[0.076923076923076, 0.076923076923076, 0.076923076923076, 0.102564102564106, 0.102564102564106, 0.102564102564106, 0.153846153846159, 0.153846153846159, 0.153846153846159],
[0.076923076923095, 0.076923076923095, 0.076923076923095,

```
    0.102564102564106, 0.102564102564106, 0.102564102564106, 0.153846153846159,
    0.153846153846159, 0.153846153846159 ]]
```