John Hermitt

2. $x'(t) = \left(\frac{5}{2}x(t)\right)\left(1 - x(t)\right)\left(1 - \frac{1}{2}x(t)\right)$

a. $0 = \left(\frac{5}{2}x\right)(1-x)\left(1 - \frac{1}{2}x\right)$

$x = 0, 1, 2$

b. $f(x) = \left(\frac{5}{2}x - \frac{5}{2}x^2\right)\left(1 - \frac{1}{2}x\right)$

$f(x) = \frac{5}{2}x - \frac{5}{4}x^2 - \frac{5}{2}x^2 + \frac{5}{4}x^3$

$f(x) = \frac{5}{2}x - \frac{15}{4}x^2 + \frac{5}{4}x^3$

$f'(x) = \frac{5}{2} - \frac{15}{2}x + \frac{15}{4}x^2$

$f'(0) = \frac{5}{2}$ unstable, not negative

$f'(1) = -\frac{5}{4}$ stable, is negative

$f'(2) = \frac{5}{2}$ unstable, not negative

3. $x(n) = \left(\frac{5}{2}x(n-1)\right)\left(1 - x(n-1)\right)\left(1 - \frac{1}{2}x(n-1)\right)$

a. $x = \frac{5}{2}x - \frac{15}{4}x^2 + \frac{5}{4}x^3$

$0 = \frac{3}{2}x - \frac{15}{4}x^2 + \frac{5}{4}x^3$

$0 = 6x - 15x^2 + 5x^3$

$0 = x(6 - 15x + 5x^2)$

$x = 0, \frac{15 \pm \sqrt{225 - 4(6)(5)}}{10}$

$x = 0, \frac{15 \pm \sqrt{105}}{10}, \frac{15 - \sqrt{105}}{10}$

b. $f'(x) = \frac{5}{2} - \frac{15}{2}x + \frac{15}{4}x^2$

$|f'(0)| = \frac{5}{2}$ unstable, not < 1

$\left|f'\left(\frac{15 + \sqrt{105}}{10}\right)\right| \approx 7.467$ unstable not < 1

$\left|f'\left(\frac{15 - \sqrt{105}}{10}\right)\right| \approx 0.217$ stable is < 1

```
> #John"John"Hermitt Final
  read "/Users/jch263/DMB.txt"
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

---

(1)

```
> read "/Users/jch263/M5.txt"
```

```
> #1
  #x(0) = 1, x(1) = 1, x(2) = 2
  #x(n) = 2·x(n-1)-x(n-3)

  x := RecToSeq([1, 1, 2], [2, 0, -1], 999)[999]:
  y := RecToSeq([1, 1, 2], [2, 0, -1], 1000)[1000]:
  evalf(y/x);
```

1.618033989 **(2)**

> #2c

$$differential := dsolve\left(\left\{diff(x(t), t) = \left(\frac{5}{2}\cdot x(t)\right)\cdot(1 - x(t))\cdot\left(1 - \frac{1}{2}\cdot x(t)\right), x(0) = 0.1\right\}, \{x(t)\}\right);$$

$$evalf(subs(t = 100, differential));$$

$$differential := x(t) = \frac{19\,e^{\frac{5t}{2}}}{81\left(-\frac{1}{\sqrt{1 + \frac{19\,e^{\frac{5t}{2}}}{81}}} - 1\right)\left(-\frac{19\,e^{\frac{5t}{2}}}{81} - 1\right)}$$

$$x(100) = 0.9999999999 \quad \textbf{(3)}$$

> #3c

$$func := \left(\frac{5}{2}\cdot x\right)\cdot(1 - x)\cdot\left(1 - \frac{1}{2}\cdot x\right):$$

$$Orb([func], [x], [0.1], 1000, 1000);$$

$$[[0.4753049232]] \quad \textbf{(4)}$$

> #4

$$Orb\left(HW3(u, v, w), [u, v, w], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], 2, 2\right);$$

$$Orb\left(HW3(u, v, w), [u, v, w], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], 1000, 1000\right)$$

$$\left[\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right]\right]$$

$$\left[\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right]\right] \quad \textbf{(5)}$$

> #5

$$M := \left[\left[\frac{1}{6}, \frac{2}{3}, \frac{1}{6}\right], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]\right]:$$

$$Orb\left(HW3g(u, v, w, M), [u, v, w], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], 2, 2\right);$$

$$OrbF\left(HW3g(u, v, w, M), [u, v, w], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], 1000, 1000\right);$$

$$\left[\left[\frac{9}{34}, \frac{1}{2}, \frac{4}{17}\right]\right]$$

$$[[0.3757562480, 0.4935706703, 0.1306730817]] \quad \textbf{(6)}$$

> #6

$$OrbF\left(\left[\frac{(1+x+y)}{2+x+3\cdot y},\frac{(1+x+3\cdot y)}{3+x+2\cdot y}\right],[x,y],[100,1000],1000,1010\right);$$

$$[[0.4705902280, 0.7478789080],[0.4705902280, 0.7478789080],[0.4705902280, 0.7478789080],[0.4705902280, 0.7478789080],[0.4705902280, 0.7478789080],[0.4705902280, 0.7478789080],[0.4705902280, 0.7478789080],[0.4705902280, 0.7478789080],[0.4705902280, 0.7478789080],[0.4705902280, 0.7478789080]] \quad (7)$$

> #7

$N := 1000:$

$v := 100:$

$g := 0.5:$

$ba := 0.05\cdot\left(\frac{v}{N}\right):$

$bb := 1.4\cdot\left(\frac{v}{N}\right):$

$bc := 1.0001\cdot\left(\frac{v}{N}\right):$

$functa := SIRS(x, y, ba, g, v, N);$

$SEquP(functa, [x, y]);$

$RemovedA := 1000 - (1000 + 0);$

$functb := SIRS(x, y, bb, g, v, N);$

$SEquP(functb, [x, y]);$

$RemovedB := 1000 - (714 + 1);$

$functc := SIRS(x, y, bc, g, v, N);$

$SEquP(functc, [x, y]);$

$$functa := [-0.005000000000\,x\,y + 500.0 - 0.5\,x - 0.5\,y,\ 0.005000000000\,x\,y - 100\,y]$$

$$\{[1000., 0.]\}$$

$$RemovedA := 0$$

$$functb := [-0.1400000000\,x\,y + 500.0 - 0.5\,x - 0.5\,y,\ 0.1400000000\,x\,y - 100\,y]$$

$$\{[714.2857143, 1.421464108]\}$$

$$RemovedB := 285$$

$$functc := [-0.1000100000\,x\,y + 500.0 - 0.5\,x - 0.5\,y,\ 0.1000100000\,x\,y - 100\,y]$$

$$\{[999.9000100, 0.0004974626915]\} \quad (8)$$

> #8

$a0 := 0:$

$aa := 1:$

$ab := 3:$

$ac1 := 7.39;$

$ac2 := 7.4;$

$B := 0.2:$

$n := 2:$

$Ga := GeneNet(a0, aa, B, n, m01, m02, m03, p01, p02, p03);$

$SEquP(Ga, [m01, m02, m03, p01, p02, p03], [m1, m2, m3, p1, p2, p3], 0.01, 10, 1);$

$Gb := GeneNet(a0, ab, B, n, m01, m02, m03, p01, p02, p03);$

```
SEquP(Gb, [m01, m02, m03, p01, p02, p03], [m1, m2, m3, p1, p2, p3], 0.01, 10, 1);
Gc1 := GeneNet(a0, ac1, B, n, m01, m02, m03, p01, p02, p03);
SEquP(Gc1, [m01, m02, m03, p01, p02, p03], [m1, m2, m3, p1, p2, p3], 0.01, 10, 1);
Gc2 := GeneNet(a0, ac2, B, n, m01, m02, m03, p01, p02, p03);
SEquP(Gc2, [m01, m02, m03, p01, p02, p03], [m1, m2, m3, p1, p2, p3], 0.01, 10, 1);
```

$$ac1 := 7.39$$

$$ac2 := 7.4$$

$$Ga := \left[-m01 + \frac{1}{p03^2 + 1}, -m02 + \frac{1}{p01^2 + 1}, -m03 + \frac{1}{p02^2 + 1}, -0.2\,p01 + 0.2\,m01, -0.2\,p02 + 0.2\,m02, -0.2\,p03 + 0.2\,m03 \right]$$

$$\{[0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038]\}$$

$$Gb := \left[-m01 + \frac{3}{p03^2 + 1}, -m02 + \frac{3}{p01^2 + 1}, -m03 + \frac{3}{p02^2 + 1}, -0.2\,p01 + 0.2\,m01, -0.2\,p02 + 0.2\,m02, -0.2\,p03 + 0.2\,m03 \right]$$

$$\{[1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663]\}$$

$$Gc1 := \left[-m01 + \frac{7.39}{p03^2 + 1}, -m02 + \frac{7.39}{p01^2 + 1}, -m03 + \frac{7.39}{p02^2 + 1}, -0.2\,p01 + 0.2\,m01, -0.2\,p02 + 0.2\,m02, -0.2\,p03 + 0.2\,m03 \right]$$

$$\{[1.777163792, 1.777163792, 1.777163792, 1.777163792, 1.777163792, 1.777163792]\}$$

$$Gc2 := \left[-m01 + \frac{7.4}{p03^2 + 1}, -m02 + \frac{7.4}{p01^2 + 1}, -m03 + \frac{7.4}{p02^2 + 1}, -0.2\,p01 + 0.2\,m01, -0.2\,p02 + 0.2\,m02, -0.2\,p03 + 0.2\,m03 \right]$$

$$\varnothing \qquad \textbf{(9)}$$

> #9

```
Eq := ChemoStat(N, C, 2.5, 2.7);
Dis(Eq, [N, C], [1, 1], 0.01, 10)[1000];
```

$$Eq := \left[\frac{2.5\,C\,N}{1 + C} - N, -\frac{C\,N}{1 + C} - C + 2.7 \right]$$

$$[10.00, [5.083016070, 0.6667368829]] \qquad \textbf{(10)}$$

> #10

```
ProbM := Matrix([[0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1], [0.1, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1,
    0.1, 0.1], [0.1, 0.1, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1], [0.075, 0.075, 0.075, 0.4, 0.075, 0.075,
    0.075, 0.075, 0.075], [0.075, 0.075, 0.075, 0.075, 0.4, 0.075, 0.075, 0.075, 0.075], [0.075,
    0.075, 0.075, 0.075, 0.075, 0.4, 0.075, 0.075, 0.075], [0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.6,
    0.05, 0.05], [0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.6, 0.05], [0.05, 0.05, 0.05, 0.05, 0.05,
```

0.05, 0.05, 0.05, 0.6]]);
$evalf\left(ProbM^{1000}\right)$;

$$ProbM := \begin{bmatrix} 0.2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.075 & 0.075 & 0.075 & 0.4 & 0.075 & 0.075 & 0.075 & 0.075 & 0.075 \\ 0.075 & 0.075 & 0.075 & 0.075 & 0.4 & 0.075 & 0.075 & 0.075 & 0.075 \\ 0.075 & 0.075 & 0.075 & 0.075 & 0.075 & 0.4 & 0.075 & 0.075 & 0.075 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.6 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.6 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.6 \end{bmatrix}$$

[[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, **(11)**
0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160,
0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801,
0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160,
0.153846153846160, 0.153846153846160],
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0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160,
0.153846153846160, 0.153846153846160],
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0.153846153846160, 0.153846153846160],
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0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160,
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[0.0769230769230801, 0.0769230769230801, 0.0769230769230801,
0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160,

```
    0.153846153846160, 0.153846153846160 ]]
>
```