Hayden Batten Final Exam Work

1) Maple

2) $x'(t) = \left(\frac{5}{2}x(t)\right)\left(1 - x(t)\right)\left(1 - \frac{1}{2}x(t)\right)$

Checked in Maple

a) $f(x) = 0 = \frac{5}{2}x(1-x)(1-\frac{1}{2}x)$ $\quad$ **$x = 0,\ 1,\ 2$ eq.pt.** $\quad$ $\frac{5}{2}x = 0$ $\quad$ $1 - x = 0$ $\quad$ $1 = \frac{1}{2}x$

b) $f'(x) = \left(\frac{5}{2}x - \frac{5}{2}x^2\right)\left(1 - \frac{1}{2}x\right) = \left(\frac{5}{2}x - \frac{5}{2}x^2 - \frac{5}{4}x^2 + \frac{5}{4}x^3\right)' = \frac{5}{2} - 5x - \frac{5}{2}x + \frac{15}{4}x^2$

$f'(x) = \frac{5}{2} - \frac{15}{2}x + \frac{15}{4}x^2$

$f'(0) = \frac{5}{2} > 0$ unstable

$f'(1) = \frac{5}{2} - \frac{15}{2} + \frac{15}{4} = -\frac{5}{4} < 0$ stable $\quad$ **$x = 1$ stable eq. pt.**

$f'(2) = \frac{5}{2} - \frac{15}{2}(2) + \frac{15}{4}(4) = \frac{5}{2} > 0$ unstable

c) Maple : **$x(100) = 0.99999999$**

3 $x(n) = \frac{5}{2}x(n-1)\left(1 - x(n-1)\right)\left(1 - \frac{1}{2}x(n-1)\right)$

a) $x = f(x)$ $\quad$ $x = \frac{5}{2}x(1-x)(1-\frac{1}{2}x) \Rightarrow x = \frac{5}{2}x - \frac{15}{4}x^2 + \frac{5}{4}x^3$

$\frac{5}{4}x^3 - \frac{15}{4}x^2 + \frac{3}{2}x = 0$ $\quad$ $x\left(\frac{5}{4}x^2 - \frac{15}{4}x + \frac{3}{2}\right) = 0$ $\quad$ **$x = 0,\ \frac{\frac{15}{4} + \sqrt{\frac{105}{16}}}{\frac{10}{4}},\ \frac{\frac{15}{4} - \sqrt{\frac{105}{16}}}{\frac{10}{4}}$ eq.pts** ($a_1$, $a_2$)

b) $f'(x) = \frac{15}{4}x^2 - \frac{15}{2}x + \frac{5}{2}$

$|f'(0)| = \frac{5}{2} > 1$ unstable

$|f'(a_1)| \approx \frac{15}{4}(a_1)^2 - \frac{15}{2}(a_1) + \frac{5}{2} \approx 7.46\ldots > 1$ unstable

$|f'(a_2)| \approx \frac{15}{4}(a_2)^2 - \frac{15}{2}(a_2) + \frac{5}{2} \approx .217 < 1$ stable

**$x = \frac{\frac{15}{4} - \sqrt{\frac{105}{16}}}{(10/4)}$ stable eq. pts**

c) Maple : **$x(1000) = 0.4753049232$** $\approx \frac{\frac{15}{4} - \sqrt{\frac{105}{16}}}{10/4}$ ✓

4) a & b) Maple

5) a & b) Maple

6) Maple

7) Maple

8) Maple

9) Maple

10) 1,2,3 0.2    4,5,6 0.4    7,8,9 0.6

$\frac{0.8}{8}$

$a = \frac{0.6}{8}$

$b = \frac{0.4}{8}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.2 | 0.1 | | | | | | | |
| 2 | 0.1 | 0.2 | 0.1 | | | | | | |
| 3 | 0.1 | 0.1 | 0.2 | 0.1 | | | | | |
| 4 | | | | 0.4 | | | | | |
| 5 | a | | | | 0.4 | | | | |
| 6 | a | | | | | 0.4 | | | |
| 7 | b | | | | | | 0.6 | | |
| 8 | b | | | | | | | 0.6 | |
| 9 | | | | | | | | | 0.6 b |

```
> #Hrudai Battini Final Exam Maple Work
  read "/Users/hb334/Documents/DMB.txt";
  read "/Users/hb334/Documents/M5.txt";
```

*First Written: Nov. 2021*

*This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)*
*accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)*

*The most current version is available on WWW at:*
*http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .*
*Please report all bugs to: DoronZeil at gmail dot com .*

*For general help, and a list of the MAIN functions,*
*type "Help();". For specific help type "Help(procedure_name);"*

*------------------------------*
*For a list of the supporting functions type: Help1();*
*For help with any of them type: Help(ProcedureName);*

*------------------------------*
*For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();*
*For help with any of them type: Help(ProcedureName);*

*------------------------------*
*For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();*
*For help with any of them type: Help(ProcedureName);*

*------------------------------* **(1)**

```
> #1
  #x(0) = 1, x(1) = 1, x(2) = 2
  #x(n) = 2*x(n-1) -x(n-3) -> recurrence equation #Day 1 is
  actually Day 0, maple numerical indexing [N+1] = Day
  DT:=RecToSeq([1,1,2], [2.,0,-1],1001)[1001]; #Population at Day
  1000
  DN:=RecToSeq([1,1,2], [2.,0,-1],1001)[1000]; #Population at Day
  999
  DIV:= (DT/DN); #The number of rabbits at Day 1000/999.
```

$$DT := 7.033036702 \times 10^{208}$$

$$DN := 4.346655726 \times 10^{208}$$

$$DIV := 1.618033989 \qquad (2)$$

```
> #2
F:= 5/2*x*(1-x)*(1-1/2*x);
#a)
EquP([F],[x]);
#b)
SEquP([F],[x]);
#c)
o:= evalf(subs(t=100,dsolve({diff(x(t),t)=(5/2*x(t))*(1-x(t))*(1
-1/2*x(t)),x(0)=0.1},{x(t)})));#Value of Equation at 100 time
stamp
```

$$F := \frac{5\,x\,(1-x)\left(1-\frac{x}{2}\right)}{2}$$

$$\{[0], [1], [2]\}$$

$$\{[1.]\}$$

$$o := x(100) = 0.9999999999 \qquad (3)$$

```
> #3
#c)
Digits:=10;
G:= 5/2*x*(1-x)*(1-1/2*x);
Orb([G],[x],[0.1],1000,1000);
```

$$Digits := 10$$

$$G := \frac{5\,x\,(1-x)\left(1-\frac{x}{2}\right)}{2}$$

$$[[0.4753049232]] \qquad (4)$$

```
> #4
#a)
Orb(HW3(x,y,z),[x,y,z],[1/3,1/3,1/3],2,2); #Proportion of Aa =
1/2 at 2nd genration
#b)
Orb(HW3(x,y,z),[x,y,z],[1/3,1/3,1/3],1000,1000); #Proportion of
Aa = 1/2 at 1000th genration
```

$$\left[\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right]\right]$$

$$\left[\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right]\right] \qquad (5)$$

```
> #5
#a)
M:= [[1,2,1],[1,1,1],[1,1,1]];
#a)#Applying the mating table as a parameter matrix from Group
5's presentaiton I use the HW3g function to return the result
when the AA female mates 2x with the Aa male and equally with the
```

```
others.
Orb(HW3g(x,y,z,M),[x,y,z],[1/3.,1/3,1/3],2,2); #Proportion of Aa
= 1/2 at 2nd genration
#b)
OrbF(HW3g(x,y,z,M),[x,y,z],[1/3.,1/3,1/3],1000,1000); #Proportion
of Aa = 1/2 at 1000th genration
```

$$M := [[1, 2, 1], [1, 1, 1], [1, 1, 1]]$$

$$[[0.3027472527, 0.4989010989, 0.1983516484]]$$

$$[[0.5512669093, 0.3974661800, 0.05126690975]] \quad (6)$$

```
> #6
OrbF([(1+x+y)/(2+x+3*y), (1+x+3*y)/(3+x+2*y)],[x,y],[100,1000],
1000,1000);
OrbF([(1+x+y)/(2+x+3*y), (1+x+3*y)/(3+x+2*y)],[x,y],[100,1000],
2000,2000);#Orbit remains the same
#I Believe y
(1000000000000000000000000000000000000000000000000000000)
would = 0.7478789080.
```

$$[[0.4705902280, 0.7478789080]]$$

$$[[0.4705902280, 0.7478789080]] \quad (7)$$

```
> #7
#a)
Fa:=SIRS(s,i,0.05*0.1,0.5,100,1000);
La:=Dis([Fa[1],Fa[2]],[s,i],[300,300],0.01,50):
La[5000];#1000-1000 = 0 removed individuals
SEquP(Fa,[s,i]); Check
#b)
Fb:=SIRS(s,i,1.4*0.1,0.5,100,1000);
Lb:=Dis([Fb[1],Fb[2]],[s,i],[300,300],0.01,50):
Lb[5000]; #1000-(714+1)=285
SEquP(Fb,[s,i]); Check
#c)NB/v -> B = v/N = B = .1  Cut off for when there would be a
non-zero number of infected people.
```

$$Fa := [-0.005\, s\, i + 500.0 - 0.5\, s - 0.5\, i,\ 0.005\, s\, i - 100\, i]$$

$$[50.00, [999.9999900, 2.499066038 \times 10^{-6580}]]$$

$$\{[1000., 0.]\}$$

$$Fb := [-0.14\, s\, i + 500.0 - 0.5\, s - 0.5\, i,\ 0.14\, s\, i - 100\, i]$$

$$[50.00, [714.2895546, 1.421192563]]$$

$$\{[714.2857143, 1.421464108]\} \quad (8)$$

```
> #8
#Help(GeneNet);
#a)
Ga:= GeneNet(0,1,0.2,2,m1,m2,m3,p1,p2,p3);
SEquP([Ga[1],Ga[2],Ga[3],Ga[4],Ga[5],Ga[6]],[m1,m2,m3,p1,p2,p3]);
#b)
Gb:= GeneNet(0,3,0.2,2,m1,m2,m3,p1,p2,p3);
SEquP([Gb[1],Gb[2],Gb[3],Gb[4],Gb[5],Gb[6]],[m1,m2,m3,p1,p2,p3]);
TimeSeries(Gb,[m1,m2,m3,p1,p2,p3],[0.2,0.1,0.3,0.1,0.4,0.5],0.01,
100,1);
TimeSeries(Gb,[m1,m2,m3,p1,p2,p3],[0.2,0.1,0.3,0.1,0.4,0.5],0.01,
```

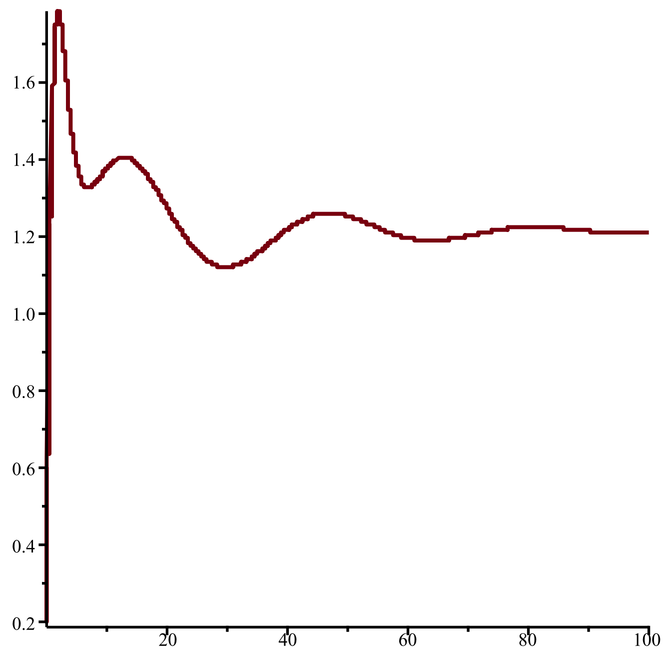
```
100,2);
TimeSeries(Gb,[m1,m2,m3,p1,p2,p3],[0.2,0.1,0.3,0.1,0.4,0.5],0.01,
100,3);
TimeSeries(Gb,[m1,m2,m3,p1,p2,p3],[0.2,0.1,0.3,0.1,0.4,0.5],0.01,
100,4);
TimeSeries(Gb,[m1,m2,m3,p1,p2,p3],[0.2,0.1,0.3,0.1,0.4,0.5],0.01,
100,5);
TimeSeries(Gb,[m1,m2,m3,p1,p2,p3],[0.2,0.1,0.3,0.1,0.4,0.5],0.01,
100,6);
#c)
Gc:= GeneNet(0,7.39,0.2,2,m1,m2,m3,p1,p2,p3);
SEquP([Gc[1],Gc[2],Gc[3],Gc[4],Gc[5],Gc[6]],[m1,m2,m3,p1,p2,p3]);
TimeSeries(Gc,[m1,m2,m3,p1,p2,p3],[0.2,0.1,0.3,0.1,0.4,0.5],0.01,
500,1);
#will take very long to reach asymptote, any longer and my
computer stalls, it shows the narrowing to an asymptote
Gco:= GeneNet(0,7.40,0.2,2,m1,m2,m3,p1,p2,p3);
SEquP([Gco[1],Gco[2],Gco[3],Gco[4],Gco[5],Gco[6]],[m1,m2,m3,p1,
p2,p3]);#No Stable EQ pt. therefore no asymptote.
```

$$Ga := \left[-m1 + \frac{1}{p3^2 + 1}, -m2 + \frac{1}{p1^2 + 1}, -m3 + \frac{1}{p2^2 + 1}, -0.2\,p1 + 0.2\,m1, -0.2\,p2 + 0.2\,m2, -0.2\,p3 + 0.2\,m3\right]$$

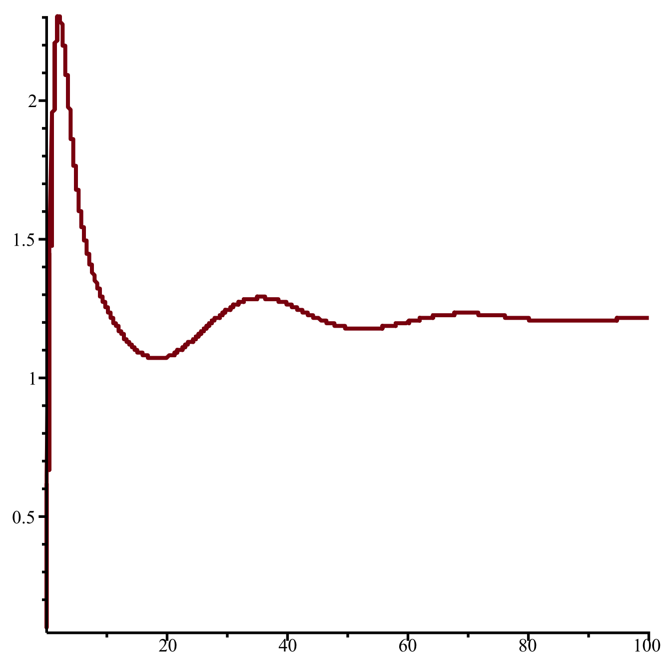
{[0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038]}

$$Gb := \left[-m1 + \frac{3}{p3^2 + 1}, -m2 + \frac{3}{p1^2 + 1}, -m3 + \frac{3}{p2^2 + 1}, -0.2\,p1 + 0.2\,m1, -0.2\,p2 + 0.2\,m2, -0.2\,p3 + 0.2\,m3\right]$$

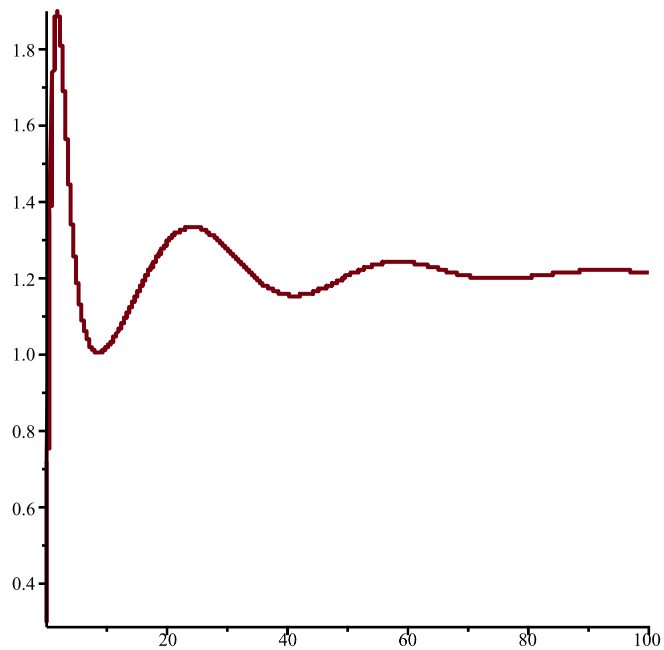
{[1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663]}

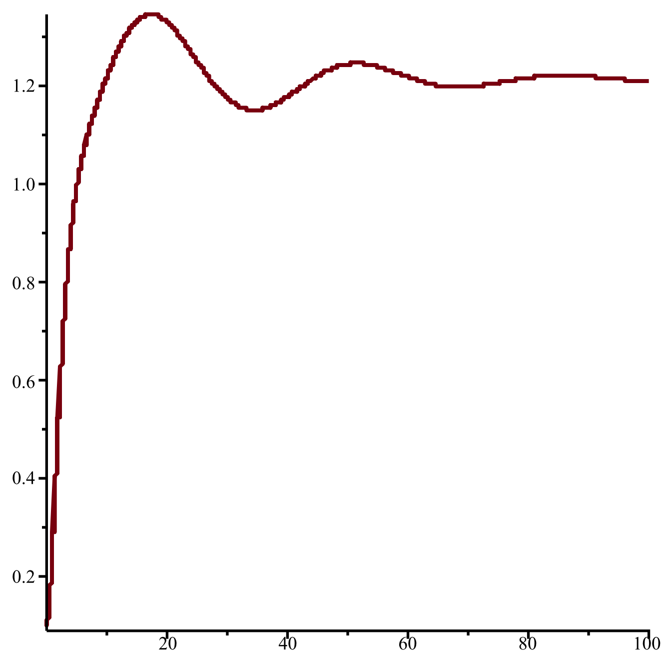


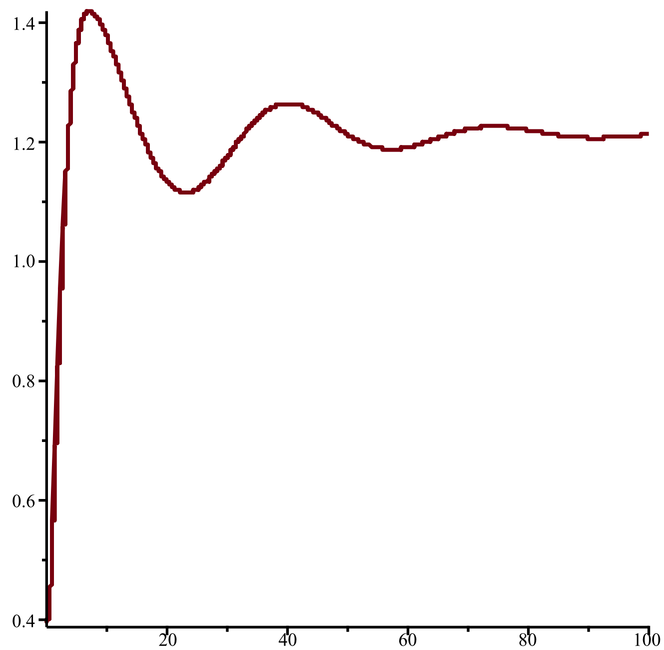
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
20
40
60
80
100

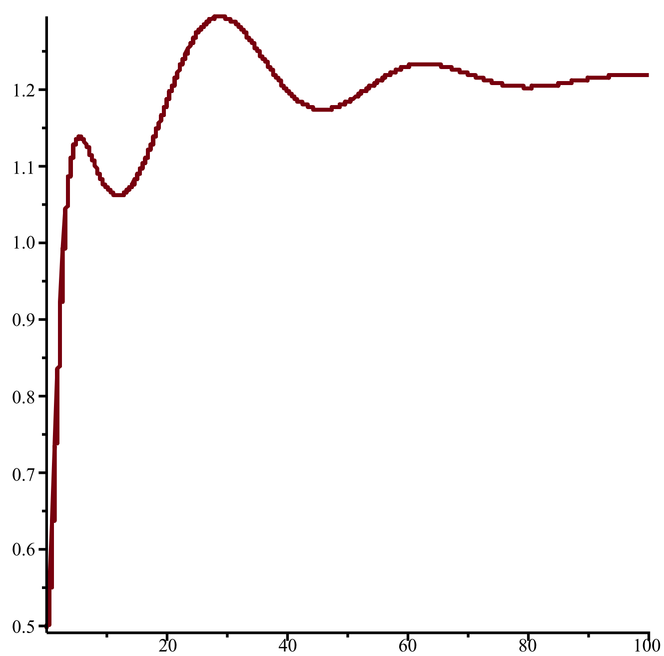



2
1.5
1
0.5
20
40
60
80
100


1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
20
40
60
80
100

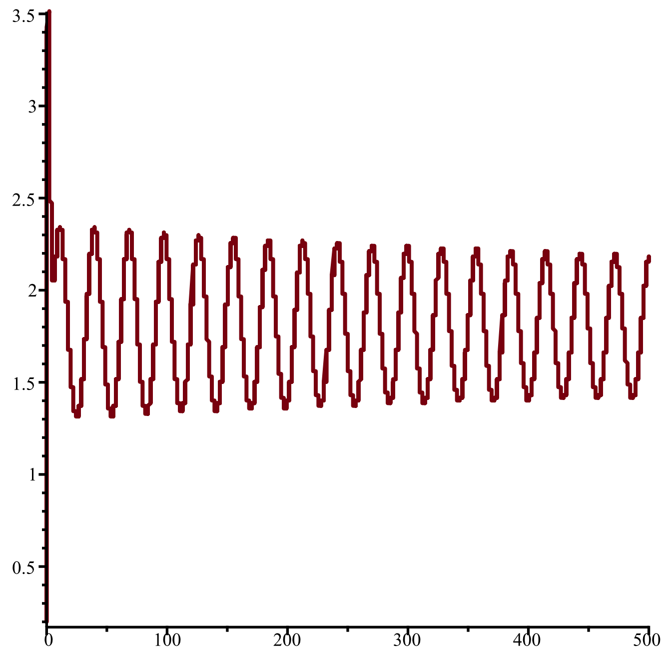

1.2
1.0
0.8
0.6
0.4
0.2
20
40
60
80
100




$$Gc := \left[ -m1 + \frac{7.39}{p3^2+1},\ -m2 + \frac{7.39}{p1^2+1},\ -m3 + \frac{7.39}{p2^2+1},\ -0.2\,p1 + 0.2\,m1,\ -0.2\,p2 + 0.2\,m2,\ -0.2\,p3 + 0.2\,m3 \right]$$

$\{[1.777163792, 1.777163792, 1.777163792, 1.777163792, 1.777163792, 1.777163792]\}$




$$Gco := \left[ -m1 + \frac{7.40}{p3^2 + 1}, -m2 + \frac{7.40}{p1^2 + 1}, -m3 + \frac{7.40}{p2^2 + 1}, -0.2\, p1 + 0.2\, m1, -0.2\, p2 + 0.2\, m2, -0.2\, p3 + 0.2\, m3 \right]$$

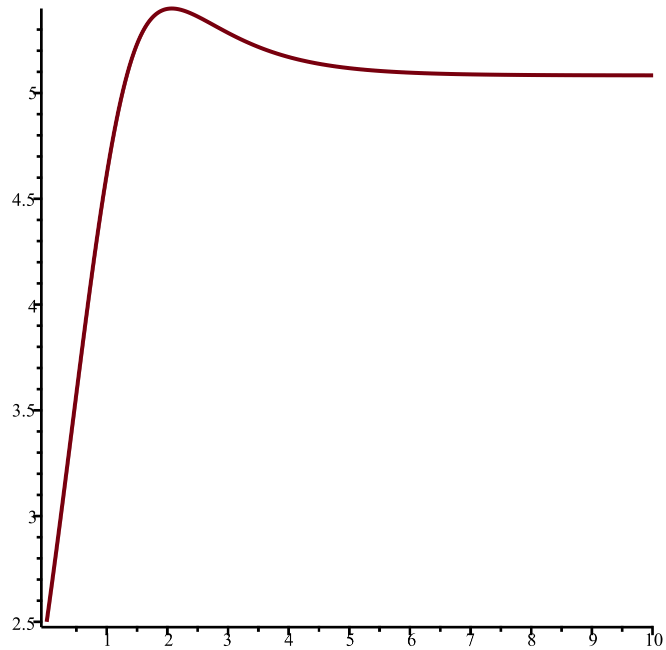
$$\emptyset \qquad (9)$$

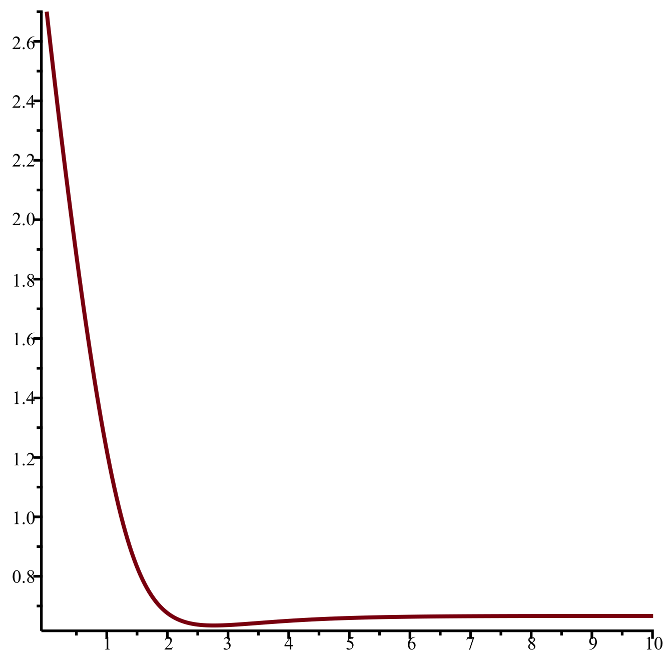
```
> #9
  K:= ChemoStat(N,C,2.5,2.7);
  KD:=Dis([K[1],K[2]], [N,C],[1,1], 0.01,50):
  KD[5001];
  SEquP([K[1],K[2]],[N,C]);#Check
  TimeSeries([K[1],K[2]],[N,C],[2.5,2.7],0.01,10,1);
  TimeSeries([K[1],K[2]],[N,C],[2.5,2.7],0.01,10,2);
```

$$K := \left[ \frac{2.5\, C\, N}{C + 1} - N, -\frac{C\, N}{C + 1} - C + 2.7 \right]$$

$$[50.01, [5.083333349, 0.6666666650]]$$

$$\{[5.083333333, 0.6666666667]\}$$




```
> #10
  Digits:= 10;
  a:=(1-0.2)/8;
  b:=(1-0.4)/8;
  c:=(1-0.6)/8;
  A:= Matrix([[0.2,a,a,a,a,a,a,a,a],
  [a,0.2,a,a,a,a,a,a,a],
  [a,a,0.2,a,a,a,a,a,a],
  [b,b,b,0.4,b,b,b,b,b],
  [b,b,b,b,0.4,b,b,b,b],
  [b,b,b,b,b,0.4,b,b,b],
  [c,c,c,c,c,c,0.6,c,c],
  [c,c,c,c,c,c,c,0.6,c],
  [c,c,c,c,c,c,c,c,0.6]]);
  A^1000;
```

$$Digits := 10$$
$$a := 0.1000000000$$
$$b := 0.07500000000$$
$$c := 0.05000000000$$

$A :=$ [[0.2, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000],
[0.1000000000, 0.2, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000],
[0.1000000000, 0.1000000000, 0.2, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000, 0.1000000000],
[0.07500000000, 0.07500000000, 0.07500000000, 0.4, 0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000],
[0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000, 0.4, 0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000],
[0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000, 0.07500000000, 0.4, 0.07500000000, 0.07500000000, 0.07500000000],
[0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.6, 0.05000000000, 0.05000000000],
[0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.6, 0.05000000000],
[0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.05000000000, 0.6]]

[[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160], **(10)**
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801,

0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160,
0.153846153846160, 0.153846153846160 ],
[ 0.0769230769230801, 0.0769230769230801, 0.0769230769230801,
0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160,
0.153846153846160, 0.153846153846160 ],
[ 0.0769230769230801, 0.0769230769230801, 0.0769230769230801,
0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160,
0.153846153846160, 0.153846153846160 ],
[ 0.0769230769230801, 0.0769230769230801, 0.0769230769230801,
0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160,
0.153846153846160, 0.153846153846160 ]]

>