> #NOT okay to post

> #Anusha Nagar, FInal Exam, 12.13.2021

>

> **read** "C://Users/an646/Documents/DMB.txt"

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

>

> #Problem 1

> #x(n) = 2·x(n-1)-x(n-3)

> #x(0)=0, x(1)=1, x(2)=2

> #I solve this problem using both RectoSeq and OrbkF

> #Define RectoSeq function that we've used before from a previous maple file

> #RecToSeq(INI,REC,N): Inputs two lists of numbers, INI and REC (of the same length, let's call it

```
    k) and a positive integer N larger than their length

    #outputs the list of the first N members of the sequence satisfying the linear recurrence with
    constants coefficients or order k
#f(n)=REC[1]*f(n-1)+... +REC[k]*f(n-k)

RecToSeq := proc(INI, REC, N) local i, k, L, newguy :
if not (type(INI, list) and type(REC, list) and nops(INI) = nops(REC) and type(N, integer)
    and N ≥ nops(INI)) then
 print(`bad innput`) :
 RETURN(FAIL) :
fi:

k := nops(INI) :

L := INI:

while nops(L) < N do
 newguy := add(REC[i]*L[-i], i=1..k) :
 L := [op(L), newguy] :
od:
L :
end:
```

```
> INI := [0, 1, 2]
```

$$INI := [0, 1, 2] \quad (2)$$

```
> REC := [2, 0,-1]
```

$$REC := [2, 0, -1] \quad (3)$$

```
> ans1 := RecToSeq(INI, REC, 1000)[1000] / RecToSeq(INI, REC, 1000)[999]
```

ans1 := (4)

351651838557114079109176274385917748850906349181793663713024525435772685590984667898711247472813058667438752246208829955440931816327251118235530060266870606369336695555990696865627993838450459511226226617017502 / 217332788434687282178442638375203129012823302586858902012408645447682777089745259452019399200396275846479612965401613173876048448116199366612355808214982204532665939691494848249642580018522380688975834246144 37

```
> ans1 := convert(ans1, float)
```

$$ans1 := 1.618033989 \quad (5)$$

```
> OrbkF(3, z, 2·z[1] + 0·z[2] − z[3], [0, 1, 2], 999, 1000)[2] / OrbkF(3, z, 2·z[1] + 0·z[2] − z[3], [0, 1, 2], 999, 1000)[1]
```

$$1.618033989 \quad (6)$$

```
> #Question 2
> #x'(t) = (2.5·x(t))·(1-x(t))·(1-0.5·x(t))
>
```

> #(a)

> $F\_2 := \left(\frac{5}{2}\cdot x\right)\cdot(1-x)\cdot\left(1-\frac{1}{2}\cdot x\right);$

$$F\_2 := \frac{5\,x\,(1-x)\left(1-\frac{x}{2}\right)}{2} \quad (7)$$

> $ans2a := EquP([F\_2],[x])$

$$ans2a := \{[0],[1],[2]\} \quad (8)$$

> #(b)

> $ans2b := SEquP([F\_2],[x])$

$$ans2b := \{[1.]\} \quad (9)$$

> #(c) Just by looking at the problem, we can see that at time 100 we would likely go to the Stable Equilibrium Point of 1. Let's confirm numerically.

> $Help(Dis)$

Dis(F,x,pt,h,A): Inputs a transformation F in the list of variables x

The approximate orbit of the Dynamical system approximating the the autonomous continuous dynamical process

dx/dt=F[1](x(t)) by a discrete time dynamical system with step-size h from t=0 to t=A

Try:

Dis([x*(1-y),y*(1-x)],[x,y],[0.5,0.5], 0.01, 10); **(10)**

> $Dis([F\_2],[x],[0.1],0.1,100)[1000]$

$$[100.0,[0.9999999996]] \quad (11)$$

> #answer to 2c: value is 1 (approaches 1 but won't ever fully reach it even if maple shows it does)

>

> #Question 3

> #x(n) = 2.5·x(n-1)·(1-x(n-1))·(1-0.5x(n-1))

> #a

> $F\_3 := \left(\frac{5}{2}\cdot x\right)\cdot(1-x)\cdot(1-0.5\cdot x)$

$$F\_3 := \frac{5\,x\,(1-x)\,(1-0.5\,x)}{2} \quad (12)$$

> $ans3a := FP([F\_3],[x])$

$$ans3a := \{[0.],[0.4753049234],[2.524695077]\} \quad (13)$$

> #b

> $ans3b := SFP([F\_3],[x])$

$$ans3b := \{[0.4753049234]\} \quad (14)$$

> #c

> #by inspection, we expect the value at day 1000 to be the SFP value of 0.4753049234 but let's confirm numerically

> $OrbF([F\_3],[x],[0.1],1000,1010)[1]$

$$[0.4753049232] \quad (15)$$

> #We are very close to approaching the stable fixed point, but with 10 decimal accuracy we are a little bit off.

>

> #Question 4

> #a

> $F\_4 := HW(u, v)$

$$F\_4 := \left[u^2 + v\,u + \frac{1}{4}\,v^2, -2\,v\,u - 2\,u^2 + 2\,u - \frac{1}{2}\,v^2 + v\right], [u, v] \quad \textbf{(16)}$$

> $Orb\left(F\_4, \left[\frac{1}{3}, \frac{1}{3}\right], 0, 2\right)$

$$\left[\left[\frac{1}{3}, \frac{1}{3}\right], \left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{1}{4}, \frac{1}{2}\right]\right] \quad \textbf{(17)}$$

> $ans4a := Orb\left(F\_4, \left[\frac{1}{3}, \frac{1}{3}\right], 0, 2\right)[2][2]$

$$ans4a := \frac{1}{2} \quad \textbf{(18)}$$

> #b

> $ans4b := Orb\left(F\_4, \left[\frac{1}{3}, \frac{1}{3}\right], 1000, 1001\right)[1][2]$

$$ans4b := \frac{1}{2} \quad \textbf{(19)}$$

> #these answers make sense bc in generalized hardy weinberg, frequencies stabilize after one generation!

> #Question 5

> #We can use HWg to set a matrix of likelihood of mating

> $mating\_matrix := [[1, 2, 1], [1, 1, 1], [1, 1, 1]]$

$$mating\_matrix := [[1, 2, 1], [1, 1, 1], [1, 1, 1]] \quad \textbf{(20)}$$

> $F\_5 := HWg(u, v, mating\_matrix) :$

> $ans5a := OrbF\left(F\_5, [u, v], \left[\frac{1}{3}, \frac{1}{3}\right], 0, 2\right)[2][2]$

$$ans5a := 0.5000000000 \quad \textbf{(21)}$$

> $ans5b := OrbF\left(F\_5, [u, v], \left[\frac{1}{3}, \frac{1}{3}\right], 1000, 1001\right)[1][2]$

$$ans5b := 0.3974661814 \quad \textbf{(22)}$$

>

> #Question 6

> $Help(Orb)$

Orb(F,x,x0,K1,K2): Inputs a transformation F in the list of variables x with initial point pt, outputs the trajectory of

of the discrete dynamical system (i.e. solutions of the difference equation): x(n)=F(x(n-1)) with x(0)=x0 from n=K1 to n=K2.

For the full trajectory (from n=0 to n=K2), use K1=0. Try:
*Orb([5/2*x*(1-x)],[x], [0.5], 1000,1010);*
*Orb([(1+x+y)/(2+x+y),(6+x+y)/(2+4*x+5*y)],[x,y], [2.,3.], 1000,1010);* **(23)**

> $\#Orb\left(\left[\frac{(1+x+y)}{(2+x+3\cdot y)}, \frac{(1+x+3\cdot y)}{(3+x+2\cdot y)}\right], [x, y], [100., 1000.], 100000000000000000000000000000000000000000000000000000, 100000000000000000000000000000000000000000000000000001\right)$

> *#I tried to run the above line of code for over 10 minutes but maple did not return an answer. I know that my syntax is correct; maple is just unable to compute the answer in a reasonable amount of time. Therefore, I will look at the stable fixed point of the system.*

> $SFP\left(\left[\frac{(1+x+y)}{(2+x+3\cdot y)}, \frac{(1+x+3\cdot y)}{(3+x+2\cdot y)}\right], [x, y]\right)$

$$\{[0.4705902280, 0.7478789082]\} \quad \textbf{(24)}$$

> *#Since we are looking for what y is going to be VERY VERY far in the future, we approach the stable fixed point likely.*

> $ans6 := SFP\left(\left[\frac{(1+x+y)}{(2+x+3\cdot y)}, \frac{(1+x+3\cdot y)}{(3+x+2\cdot y)}\right], [x, y]\right)[1][2]$

$$ans6 := 0.7478789082 \quad \textbf{(25)}$$

> *#Question 7*

> *#a*

> $F\_7a := Dis(SIRS(s, i, 0.05, 0.5, 100, 1000), [s, i], [300, 300], 0.01, 10)[1000]$

$$F\_7a := [10.00, [994.9597399, 4.087387574 \times 10^{-384}]] \quad \textbf{(26)}$$

> $ans7a := 1000 - F\_7a[2][1] - F\_7a[2][2]$

$$ans7a := 5.0402601 \quad \textbf{(27)}$$

> $Digits := 10$

$$Digits := 10 \quad \textbf{(28)}$$

> $F\_7b := Dis(SIRS(s, i, 1.4, 0.5, 100, 1000), [s, i], [300, 300], 0.01, 10)[1000]$

$$F\_7b := [10.00, [\mathrm{Float}(\infty), \mathrm{Float}(-\infty)]] \quad \textbf{(29)}$$

> $1000 - F\_7b[2][1] - F\_7b[2][2]$

$$\mathrm{Float(undefined)} \quad \textbf{(30)}$$

> *#c - we expect cut-off to be where beta < nu divided by N, as this is where number of infected goes to 0. We'll investigate around nu over N, or 0.1 as beta*

> $F\_7c1 := Dis(SIRS(s, i, 0.1, 0.5, 100, 1000), [s, i], [300, 300], 0.01, 10)[1000]$

$$F\_7c1 := [10.00, [994.5393771, 6.460506497 \times 10^{-94}]] \quad \textbf{(31)}$$

> $F\_7c2 := Dis(SIRS(s, i, 0.98, 0.5, 100, 1000), [s, i], [300, 300], 0.01, 10)[1000]$

$$F\_7c2 := [10.00, [\mathrm{Float}(\infty), \mathrm{Float}(-\infty)]] \quad \textbf{(32)}$$

> $F\_7c3 := Dis(SIRS(s, i, 0.11, 0.5, 100, 1000), [s, i], [300, 300], 0.01, 10)[1000]$

$$F\_7c3 := [10.00, [994.4514918, 6.386636089 \times 10^{-55}]] \quad \textbf{(33)}$$

> $F\_7c5 := Dis(SIRS(s, i, 0.12, 0.5, 100, 1000), [s, i], [300, 300], 0.01, 10)[1000]$

(34)

$$F\_7c5 := [10.00, [994.3637683, 1.380574328 \times 10^{-19}]] \quad (34)$$

> F_7c6 := Dis(SIRS(s, i, 0.13, 0.5, 100, 1000), [s, i], [300, 300], 0.01, 10)[1000]

$$F\_7c6 := [10.00, [732.5758779, 3.257054063 \times 10^{-6}]] \quad (35)$$

> F_7c7 := Dis(SIRS(s, i, 0.14, 0.5, 100, 1000), [s, i], [300, 300], 0.01, 10)[1000]

$$F\_7c7 := [10.00, [750.5999123, 15.61186610]] \quad (36)$$

> F_7c8 := Dis(SIRS(s, i, 0.135, 0.5, 100, 1000), [s, i], [300, 300], 0.01, 10)[1000]

$$F\_7c8 := [10.00, [828.1171053, 0.00004338056080]] \quad (37)$$

> F_7c9 := Dis(SIRS(s, i, 0.136, 0.5, 100, 1000), [s, i], [300, 300], 0.01, 10)[1000]

$$F\_7c9 := [10.00, [840.3067150, 0.0003801129174]] \quad (38)$$

> F_7c10 := Dis(SIRS(s, i, 0.132, 0.5, 100, 1000), [s, i], [300, 300], 0.01, 10)[1000]

$$F\_7c10 := [10.00, [779.6335148, 1.057669494 \times 10^{-6}]] \quad (39)$$

> F_7c11 := Dis(SIRS(s, i, 0.133, 0.5, 100, 1000), [s, i], [300, 300], 0.01, 10)[1000]

$$F\_7c11 := [10.00, [798.1735564, 2.067589402 \times 10^{-6}]] \quad (40)$$

> F_7c12 := Dis(SIRS(s, i, 0.134, 0.5, 100, 1000), [s, i], [300, 300], 0.01, 10)[1000]

$$F\_7c12 := [10.00, [814.1882691, 7.407152249 \times 10^{-6}]] \quad (41)$$

> *#even though we assumed cut-off to be around 0.1, it actually is at 0.135 (where we are no longer extremely close to 0).*

> *#Problem 8*

> Help(GeneNet)

GeneNet(a0,a,b,n,m1,m2,m3,p1,p2,p3): The contiuous-time dynamical system, with quantities m1,m2,m3,p1,p2,p3, due to M. Elowitz and S. Leibler

described in the Ellner-Guckenheimer book, Eq. (4.1) (chapter 4, p. 112)

and parameers a0 (called alpha_0 there),a (called alpha there), b (called beta there) and n.

Try:

GeneNet(0,0.5,0.2,2,m1,m2,m3,p1,p2,p3); **(42)**

> F_8 := GeneNet(0, 1, 0.2, 2, m1, m2, m3, p1, p2, p3)

$$F\_8 := \left[-m1 + \frac{1}{p3^2+1}, -m2 + \frac{1}{p1^2+1}, -m3 + \frac{1}{p2^2+1}, -0.2\,p1 + 0.2\,m1, -0.2\,p2 + 0.2\,m2, -0.2\,p3 + 0.2\,m3\right] \quad (43)$$

> *#a*

> ans8a := SEquP(F_8, [m1, m2, m3, p1, p2, p3])

$$ans8a := \{[0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038, 0.6823278038]\} \quad (44)$$

>

> *#b*

> F_8b := GeneNet(0, 3, 0.2, 2, m1, m2, m3, p1, p2, p3)

$$F\_8b := \left[-m1 + \frac{3}{p3^2+1}, -m2 + \frac{3}{p1^2+1}, -m3 + \frac{3}{p2^2+1}, -0.2\,p1 + 0.2\,m1, -0.2\,p2 \right. \quad (45)$$

$+ 0.2\, m2,\ -0.2\, p3 + 0.2\, m3]$

> ans8b := SEquP(F_8b, [m1, m2, m3, p1, p2, p3])

ans8b := {[1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663, 1.213411663]} **(46)**

>

> #c

> SEquP(GeneNet(0, 50, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

∅ **(47)**

> #Confirmed as there is no stable equilibrium point

> SEquP(GeneNet(0, 20, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

∅ **(48)**

> SEquP(GeneNet(0, 10, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

∅ **(49)**

> SEquP(GeneNet(0, 5, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

{[1.515980228, 1.515980228, 1.515980228, 1.515980228, 1.515980228, 1.515980228]} **(50)**

> #alpha has to be between 5 and 10

> SEquP(GeneNet(0, 6, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

{[1.634365293, 1.634365293, 1.634365293, 1.634365293, 1.634365293, 1.634365293]} **(51)**

> SEquP(GeneNet(0, 7, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

{[1.739203861, 1.739203861, 1.739203861, 1.739203861, 1.739203861, 1.739203861]} **(52)**

> SEquP(GeneNet(0, 8, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

∅ **(53)**

> SEquP(GeneNet(0, 7.5, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

∅ **(54)**

> SEquP(GeneNet(0, 7.4, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

∅ **(55)**

> SEquP(GeneNet(0, 7.3, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

{[1.768534008, 1.768534008, 1.768534008, 1.768534008, 1.768534008, 1.768534008]} **(56)**

> SEquP(GeneNet(0, 7.35, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

{[1.773337706, 1.773337706, 1.773337706, 1.773337706, 1.773337706, 1.773337706]} **(57)**

> SEquP(GeneNet(0, 7.36, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

{[1.774295627, 1.774295627, 1.774295627, 1.774295627, 1.774295627, 1.774295627]} **(58)**

> SEquP(GeneNet(0, 7.37, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

{[1.775252614, 1.775252614, 1.775252614, 1.775252614, 1.775252614, 1.775252614]} **(59)**

> SEquP(GeneNet(0, 7.38, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

{[1.776208668, 1.776208668, 1.776208668, 1.776208668, 1.776208668, 1.776208668]} **(60)**

> SEquP(GeneNet(0, 7.39, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

{[1.777163792, 1.777163792, 1.777163792, 1.777163792, 1.777163792, 1.777163792]} **(61)**

> SEquP(GeneNet(0, 7.40, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

∅ **(62)**

> #7.39 is the alpha such that there is still a stable equilibrium. Once we get to alpha = 7.4, we no longer have a stable equilibrium

>

>

>

> #Question 9

> Help(ChemoStat)

ChemoStat(N,C,a1,a2): The Chemostat continuous-time dynamical system with N=Bacterial poplulation densitty, and C=nutient Concentration in growth chamber (see Table 4.1 of Edelstein-Keshet, p. 122)

with paramerts a1, a2, Equations (19a_, (19b) in Edelestein-Keshet p. 127 (section 4.5, where they are called alpha1, alpha2). a1 and a2 can be symbolic or numeric. Try:

ChemoStat(N,C,a1,a2);

ChemoStat(N,C,2,3); **(63)**

> F_9 := ChemoStat(N, C, 2.5, 2.7)

$$F\_9 := \left[\frac{2.5\, C\, N}{C+1} - N, -\frac{C\, N}{C+1} - C + 2.7\right] \tag{64}$$

> Dis(F_9, [N, C], [0.5, 0.5], 0.01, 10)[1000]

[10.00, [5.082813723, 0.6667872951]] **(65)**

> ans9a := Dis(F_9, [N, C], [0.5, 0.5], 0.01, 10)[1000][2][1]

ans9a := 5.082813723 **(66)**

> ans9b := Dis(F_9, [N, C], [0.5, 0.5], 0.01, 10)[1000][2][2]

ans9b := 0.6667872951 **(67)**

>

>

> #Question 10

>

> page_surfer_matrix := Matrix([[0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1], [0.1, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1], [0.1, 0.1, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1], [0.075, 0.075, 0.075, 0.4, 0.075, 0.075, 0.075, 0.075, 0.075], [0.075, 0.075, 0.075, 0.075, 0.4, 0.075, 0.075, 0.075, 0.075], [0.075, 0.075, 0.075, 0.075, 0.075, 0.4, 0.075, 0.075, 0.075], [0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.6, 0.05, 0.05], [0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.6, 0.05], [0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.6]])

(68)

$$page\_surfer\_matrix := \begin{bmatrix} 0.2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.075 & 0.075 & 0.075 & 0.4 & 0.075 & 0.075 & 0.075 & 0.075 & 0.075 \\ 0.075 & 0.075 & 0.075 & 0.075 & 0.4 & 0.075 & 0.075 & 0.075 & 0.075 \\ 0.075 & 0.075 & 0.075 & 0.075 & 0.075 & 0.4 & 0.075 & 0.075 & 0.075 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.6 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.6 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.6 \end{bmatrix} \quad \textbf{(68)}$$

> $page\_surfer\_matrix^{1000}$

[[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160], **(69)**
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160,

0.153846153846160, 0.153846153846160]]

>

> *#a*

> *#Probability of web-page 1 in the long run: 0.0769230769230801*

> *#b*

> *#Probability of web-page 9 in the long run: 0.153846153846160*

>

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