Anne Somalwar

1. $x(0) = 1$, $x(1) = 1$, $x(2) = 2$

$$x(n) = 2x(n-1) - x(n-3)$$

Used Orbk(3, z, 2*z[1] - z[3], [1.,1.,2.],
999, 1000);

and got $[2.686 \times 10^{208}, 4.347 \times 10^{208}]$

Then, $\frac{4.347}{2.686} = \boxed{1.618}$.

2) $\frac{dx}{dt} = (5/2\, x)(1-x)(1 - 1/2\, x)$

a) $(5/2\, x)(1-x)(1-1/2\, x) = 0$

$\Rightarrow \boxed{x = 0,\ 1,\ 2}$

Checked with EquP([(5/2 * x) * (1-x) * (1-(1/2)*x)],
[x]);

b) $F(x) = (5/2\, x)(1-x)(1-1/2\, x)$

$= \left(5/2\, x - 5/2\, x^2\right)(1 - 1/2\, x)$

$= 5/2\, x - 5/4\, x^2 - 5/2\, x^2 + 5/4\, x^3$

$F'(x) = 5/2 - 5/2\, x - 5x + 15/4\, x^2$

$F'(0) = \frac{5}{2} > 0$, so 0 is <u>not</u> stable.

$F'(1) = \cancel{\frac{5}{2} - \frac{5}{2}} - 5 + \frac{15}{4} = \frac{15}{4} - 5 < 0$ so

1 is stable.

$F'(2) = \frac{5}{2} - \cancel{5 - 10 + 15} = \frac{5}{2} > 0$ so

2 is <u>not</u> stable.

Checked with

```
SEquP([((5/2)*x)*(1-x)*(1-(1/2)*x)],
      [x]);
```

c) $x(0) = 0.1$

$$\boxed{\text{Approximately } 0.9999\ldots}$$

Checked with

```
dsolve([diff(x(t),t) = ((5/2)*x)*(1-x)*(1-(1/2)*x),
        x(0)=0.1], [x(t)]);
```

plugging in $t=100$

3. $x(n) = \left(\frac{5}{2}x(n-1)\right)(1 - x(n-1))\left(1 - \frac{1}{2}x(n-1)\right)$

a) Used

FP([((5/2)*x)*(1-x)*(1-(1/2)*x)], [x]);

$\Rightarrow \left\{[0], \left[\frac{3}{2} - \frac{\sqrt{105}}{10}\right], \left[\frac{3}{2} + \frac{\sqrt{105}}{10}\right]\right\}$

b) Used

SFP([((5/2)*x)*(1-x)*(1-(1/2)*x)], [x]);

$\Rightarrow \left[\frac{3}{2} - \frac{\sqrt{105}}{10}\right] \approx [0.475]$

3c)

Used Orb([((5/2)*x)*(1-x)*(1-(y/2)*x)], [x],

[.1], 1000, 1000)

$\Rightarrow$ 0.4753049232

4. (a) $u = 1/3$, $v = 1/3$, $w = 1/3$

$$v_{n+1} = u_n v_n + 2u_n w_n + \tfrac{1}{2} v_n^2 + v_n w_n$$

$$v_1 = (1/3)(1/3) + 2(1/3)(1/3) + 1/2\,(1/3)^2 + (1/3)(1/3)$$

$$\boxed{= 0.5}$$

Checked with

HW(1/3, 1/3);

b) $\boxed{1/2}$ (stabilizes after one gen)

5. a) Used

HWg(1/3, 1/3, [[1,2,1], [1,1,1], [1,1,1]]);

$\boxed{1/2}$

b) Used

F:= HWg(u, v, [[1,2,1], [1,1,1], [1,1,1]]);

OrbF(F, [u,v], [1/3, 1/3], 999, 1000);

$\Rightarrow \boxed{0.3974661814}$

6. Used

SFP([(1+x+y)/(2+x+3*y),

(1+x+3*y)/(3+x+2*y)], [x,y])

$\Rightarrow$ [0.4705902280, 0.7478789082]

Then,

OrbF([(1+x+y)/(2+x+3*y),

(1+x+3*y)/(3+x+2*y)], [x,y],

[100., 100.], 1000, 1010)

$\Rightarrow$ [[0.4705902280, 0.7478789082] ...]

so

$y(100\ldots)$

≈ 0.7478789082

7.

a) $R_0 = \frac{N\beta}{\nu} = \frac{1000 \cdot 0.05}{100} = \frac{1}{2} \leq 1$

so there are $\boxed{0}$ removed in the long run

b) $R_0 > 1$, so

$$S = \frac{v}{\beta} = \frac{100}{1.4} = 71.43$$

$$I = \gamma \frac{N-S}{v+\gamma} = 0.5\left(\frac{1000 - 71.43}{100 + 0.5}\right) = 4.6198$$

$$R = 1000 - 71.43 - 0.1421 = \boxed{923.95}$$

c) $R_0 = \frac{N\beta}{v} = \frac{1000\beta}{100} = 1$

$$10\beta = 1 \Rightarrow \boxed{\beta = 1/10}$$

8. a)

```
F:= GeneNet(0,1,0.2,2,m1,m2,m3,p1,p2,p3);
SEquP(F,[m1,m2,m3,p1,p2,p3]);
```

$\Rightarrow$ $\boxed{0.6823278038}$

b)

```
F:= GeneNet(0,3,0.2,2,m1,m2,m3,p1,p2,p3);
SEquP(F,[m1,m2,m3,p1,p2,p3]);
```

$\Rightarrow$ $\boxed{\text{yes},\quad 1.213411663}$

c)

F:= GeneNet(0, 7.39, 0.2, 2, m1, m2, m3, p1, p2, p3);

SEquP(F, [m1, m2, m3, p1, p2, p3]);

$\Rightarrow$ [1.777163792, 1.777163792, ...]

F:= GeneNet(0, 7.4, 0.2, 2, m1, m2, m3, p1, p2, p3);

SEquP(F, [m1, m2, m3, p1, p2, p3]);

$\Rightarrow \emptyset$

$\boxed{\alpha = 4.39}$

9.

a) F := ChemoStat(N, C, 2.5, 2.7);

SEquP(F, [N, C]);

$\Rightarrow$ {[5.083333, 0.66667]}

$\boxed{5.083333}$

b) $\boxed{0.66667}$

(unless there are no bacteria to start with)

10.

	1	2	3	4	5	6	7	8	9
1	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
2	0.1	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1
3	0.1	0.1	0.2	0.1	0.1	0.1	0.1	0.1	0.1
4	0.075	0.075	0.075	0.4	0.075	0.075	0.075	0.075	0.075
5	0.075	0.075	0.075	0.075	0.4	0.075	0.075	0.075	0.075
6	0.075	0.075	0.075	0.075	0.075	0.4	0.075	0.075	0.075
7	0.05	0.05	0.05	0.05	0.05	0.05	0.6	0.05	0.05
8	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.6	0.05
9	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.6

Using the above matrix as M,

$$M^{100} = \begin{pmatrix} 0.0769 & 0.0769 & 0.0769 & 0.1026 & 0.1026 & 0.1538 & 0.1538 & 0.1538 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

a) $\boxed{0.076923}$ (7.69%)

b) $\boxed{0.153846}$ (15.38%)