Final - Alan Ho

1)

$x(0)=1 \quad x(1)=1 \quad x(2)=2 \quad x(3)=2(2)-1=3$

$x(n) = 2(x(n-1)) - x(n-3)$

$\frac{x(1000)}{x(999)} = \frac{\text{Orbk}(3, z, (2\cdot z[1] - z[3]), [1,1,2], 100, 1000)[1]}{\text{Orbk}(3, z, (2\cdot z[1] - z[3]), [1,1,2], 999, 999)[1]} = \boxed{1.618}$

Transformation: $x_1(n) = 2 \cdot x_1(n-1) - x_3(n-1)$

~~[illegible]~~

2)

$x'(t) = \frac{5}{2} x(t)(1-x(t))\left(1-\frac{x(t)}{2}\right)$

a) $0 = \frac{5x}{2}(1-x)\left(1-\frac{x}{2}\right) \qquad f(x) = \frac{5x}{2}(1-x)\left(1-\frac{x}{2}\right)$

$\boxed{x = 0, 1, 2}$

b) $f'(x) = \text{diff}\left(\frac{5x}{2}\cdot(1-x)\cdot\left(1-\frac{x}{2}\right), x\right) = \frac{5(1-x)(1-\frac{x}{2})}{2} - \frac{5x(1-\frac{x}{2})}{2} - \frac{5x(1-x)}{4}$

$f'(0) = \text{subs}(x=0, \%) = 2.5 > 0 \therefore$ unstable

$f'(1) = \text{subs}(x=1, \%) = -\frac{5}{4} < 0 \therefore$ $\boxed{\text{stable}}$

$f'(2) = \text{subs}(x=2, \%) = \frac{5}{2} > 0 \therefore$ unstable

c) $x(0) = 0.1$

approaches 1 b/c it is stable

evalf(subs(t=100, dsolve({diff(x(t),t) = 5/2 · x(t) · (1-x(t)) · (1 - 1/2 · x(t)), x(0) = 0.1}, x(t)))) = x(100) = 0.9999

3)

$x(n) = \frac{5}{2} x(n-1)(1-x(n-1))\left(1-\frac{x(n-1)}{2}\right)$

a) $f(x) = \frac{5}{2}x(1-x)\left(1-\frac{x}{2}\right)$

$x = \frac{5x}{2}(1-x)\left(1-\frac{x}{2}\right)$: evalf(FP([5x/2 · (1-x) · (1-x/2)], [x]))

$= \boxed{\{[0], [0.475], [2.525]\}}$

b) evalf(SFP([5x/2 · (1-x) · (1-x/2)], [x]))

$\boxed{= \{[0.4753]\}}$

c) $x(0) = 0.1$

x(100) = Orb([5x/2 · (1-x) · (1-x/2)], [x], [0.1], 100, 100)[1] = $\boxed{0.4753}$

4) $p^2 = AA$, $2pq = Aa$, $q^2 = aa$

$p^2 + 2pq + q^2 = 1$ $\therefore p^2 = 1/3$, $2pq = 1/3$, $q^2 = 1/3$

a) If the population is in Hardy-Weinberg equilibrium, then frequency distribution will not change over generations $\therefore$ the proportion of the 2nd generation w/ Aa will be still 1/3

b) Frequency does not change among generations in H/W equilibrium $\therefore$ 100th generation will still 1/3 of its population as Aa

5) 4+5) see attached

6) $x(n) = \dfrac{1 + x(n-1) + y(n-1)}{2 + x(n-1) + 3y(n-1)}$ $y(n) = \dfrac{1 + x(n-1) + 3y(n-1)}{3 + x(n-1) + 2y(n-1)}$

$(x,y) \to \left(\dfrac{1+x+y}{2+x+3y}, \dfrac{1+x+3y}{3+x+2y}\right)$

$SFP\left(\left[\dfrac{1+x+y}{2+x+3y}, \dfrac{1+x+3y}{3+x+2y}\right], [x,y]\right) = [0.47059, 0.74787]$

Orb (F, [x,y], [100., 1000.], 1000, 1010) = ↓

$\therefore$ $y(1.0 \times 10^n)$ will always $= \boxed{0.74787890}$

7) $N=1000$, $\gamma=0.5$, $r=100$, $s(0)=300$, $i(0)=300$

a) SIRS(s, i, 0.05, 0.5, 100, 1000) = [−0.05si + 500 − 0.5s − 0.5i, 0.05si − 100i]

SEquP(↲ , [s,i]) = ([1,000, 0])

removed = $N - S - i$ = 1,000 − 1,000 − 0 = **[0]**

b) SIRS(s, i, 1.4, 0.5, 100, 1000) = [−1.4si + 500 − 0.5s − 0.5i, 1.4si − 100i]

SEquP(↲ , [s,i]) = [71.4, 4.619]

1,000 − 71.4 − 4.6 = **[923.95]**

c) somewhere b/w 0.05 and 1.4

[See attached maple pdf] bifurcation is at $\beta = 0.1$

8) a) • SEquP(GeneNet(0.1, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3])

= {[0.682, ⟵ , , , ⟶, 0.682]} = **[0.6823]**

confirmed via TimeSeries for i = 1...6

b) • TimeSeries(GeneNet(0.3, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3], [0,0,0,0,0,0], 0.1, 100, i)

for i = 1...6, there exists a horizontal asymptote!

• SEquP(GeneNet(0.3, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1 .... p3])

= {[1.213 .... 1.213]} = ~~0.12134~~ **[1.2134]**

c) ~~[see maple]~~

9) • SEquP(ChemoStat(N, C, 2.5, 2.7), [N, C])

= ~~[N~~ {[5.08, 0.667]}

[N = 5.0833 C = 0.6667]

[see attached]

10)

$M :=$

	1	2	3	4	5	6	7	8	9
1	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
2	0.1	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1
3	0.1	0.1	0.2	0.1	0.1	0.1	0.1	0.1	0.1
4	0.075	0.075	0.075	0.4	0.075	0.075	0.075	0.075	0.075
5	0.075	0.075	0.075	0.075	0.4	0.075	0.075	0.075	0.075
6	0.075	0.075	0.075	0.075	0.075	0.4	0.075	0.075	0.075
7	0.05	0.05	0.05	0.05	0.05	0.05	0.6	0.05	0.05
8	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.6	0.05
9	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.6

$M^{1000} = [[0.0769, 0.0769, 0.0769, 0.102, 0.102, 0.102, 0.153, 0.153, 0.153]$
$[\ldots\ldots\ldots]]$

11) ~~$x = x = x = 1/3$~~

4)

```
> read("DMB.txt")
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

$$> \ Orb\left(\left(HW3(u, v, w), [u, v, w], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], 1, 1\right)\right)$$

$$\left[\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right]\right] \quad \textbf{(2)}$$

$\frac{1}{2}$ *is the proportion of the population that has Aa*

$$> \ Orb\left(\left(HW3(u, v, w), [u, v, w], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], 999, 999\right)\right)$$

$$\left[\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right]\right] \quad (3)$$

5)

> with(LinearAlgebra)

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip] **(4)**

> $M := Matrix([[0.1, 0.2, 0.1], [0.1, 0.1, 0.1], [0.1, 0.1, 0.1]])$

$$M := \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} \quad (5)$$

> $Orb\left(HW3g(u, v, w, M), [u, v, w], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], 1, 1\right)$

$$[[0.2750000001, 0.5000000001, 0.2250000000]] \quad (6)$$

> $Orb\left(HW3g(u, v, w, M), [u, v, w], \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], 999, 999\right)$

$$[[0.5512669096, 0.3974661806, 0.05126690988]] \quad (7)$$

7 c)

```
> read("DMB.txt")
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

```
> SIRS(s, i, beta, 0.5, 100, 1000)
```

$$[-s\,i\,\beta + 500.0 - 0.5\,s - 0.5\,i,\ s\,i\,\beta - 100\,i] \quad \textbf{(2)}$$

```
> sys := {-s i beta + 500.0 - 0.5 s - 0.5 i = 0, s i beta - 100 i = 0}
```

$$sys := \{-s\,i\,\beta + 500.0 - 0.5\,s - 0.5\,i = 0,\ s\,i\,\beta - 100\,i = 0\} \quad \textbf{(3)}$$

```
> solve(sys, {s, i})
```

$$\{i = 0.,\ s = 1000.\},\ \left\{i = \frac{0.4975124378\,(10.\,\beta - 1.)}{\beta},\ s = \frac{100.}{\beta}\right\} \quad \textbf{(4)}$$

> $solve\left(1000 = \frac{100}{\text{beta}}, \text{beta}\right)$

$$\frac{1}{10} \tag{5}$$

> $solve\left(0 = \frac{0.4975124378\ (10.\ \text{beta} - 1.)}{a}, \text{beta}\right)$

$$0.1000000000 \tag{6}$$

>

8 c)

```
> read("DMB.txt")
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

```
> GeneNet((0, a, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3], [0, 0, 0, 0, 0, 0], 0.1, 100, 1)
```

$$\left[-m1+\frac{a}{p3^2+1},\ -m2+\frac{a}{p1^2+1},\ -m3+\frac{a}{p2^2+1},\ -0.2\,p1+0.2\,m1,\ -0.2\,p2+0.2\,m2,\ -0.2\,p3+0.2\,m3\right] \quad \textbf{(2)}$$

```
> M := JAC(GeneNet((0, a, 0.2, 2, m1, m2, m3, p1, p2, p3), [m1, m2, m3, p1, p2, p3], [0, 0, 0, 0, 0, 0], 0.1, 100, 1), [m1, m2, m3, p1, p2, p3])
```

(3)

$$M := \left[\left[-1, 0, 0, 0, 0, -\frac{2\,a\,p3}{\left(p3^2+1\right)^2}\right], \left[0, -1, 0, -\frac{2\,a\,p1}{\left(p1^2+1\right)^2}, 0, 0\right], \left[0, 0, -1, 0, -\frac{2\,a\,p2}{\left(p2^2+1\right)^2}, 0\right], [0.2, 0, 0, -0.2, 0, 0], [0, 0.2, 0, 0, -0.2, 0], [0, 0, 0.2, 0, 0, -0.2]\right] \tag{3}$$

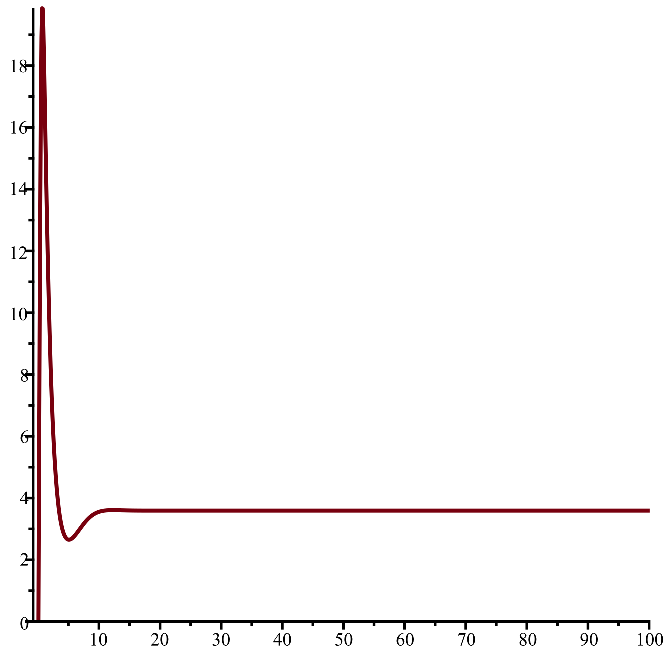
> *SEquP*(*GeneNet*((0, 8, 0.2, 2, *m1*, *m2*, *m3*, *p1*, *p2*, *p3*), [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [0.2, 0.1, 0.3, 0.1, 0.4, 0.5], 0.1, 100, 1), [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*])

$$\varnothing \tag{4}$$

> *EquP*(*GeneNet*((0, 8, 0.2, 2, *m1*, *m2*, *m3*, *p1*, *p2*, *p3*), [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [0.2, 0.1, 0.3, 0.1, 0.4, 0.5], 0.1, 100, 1), [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*])

{[1.833750958, 1.833750958, 1.833750958, 1.833750958, 1.833750958, 1.833750958], **(5)**
[−1.671625279 − 0.9400455239 I, 1.269008672 − 1.370228794 I, 0.4641550685 + 2.202582010 I, −1.671625279 − 0.9400455239 I, 1.269008672 − 1.370228794 I, 0.4641550685 + 2.202582010 I], [−1.671625279 + 0.9400455239 I, 1.269008672 + 1.370228794 I, 0.4641550685 − 2.202582010 I, −1.671625279 + 0.9400455239 I, 1.269008672 + 1.370228794 I, 0.4641550685 − 2.202582010 I], [−0.9168754789 − 1.876694416 I, −0.9168754789 − 1.876694416 I, −0.9168754789 − 1.876694416 I, −0.9168754789 − 1.876694416 I, −0.9168754789 − 1.876694416 I, −0.9168754789 − 1.876694416 I], [−0.9168754789 + 1.876694416 I, −0.9168754789 + 1.876694416 I, −0.9168754789 + 1.876694416 I, −0.9168754789 + 1.876694416 I, −0.9168754789 + 1.876694416 I, −0.9168754789 + 1.876694416 I], [0.4641550685 − 2.202582010 I, −1.671625279 + 0.9400455239 I, 1.269008672 + 1.370228794 I, 0.4641550685 − 2.202582010 I, −1.671625279 + 0.9400455239 I, 1.269008672 + 1.370228794 I], [0.4641550685 + 2.202582010 I, −1.671625279 − 0.9400455239 I, 1.269008672 − 1.370228794 I, 0.4641550685 + 2.202582010 I, −1.671625279 − 0.9400455239 I, 1.269008672 − 1.370228794 I], [1.269008672 − 1.370228794 I, 0.4641550685 + 2.202582010 I, −1.671625279 − 0.9400455239 I, 1.269008672 − 1.370228794 I, 0.4641550685 + 2.202582010 I, −1.671625279 − 0.9400455239 I], [1.269008672 + 1.370228794 I, 0.4641550685 − 2.202582010 I, −1.671625279 + 0.9400455239 I, 1.269008672 + 1.370228794 I, 0.4641550685 − 2.202582010 I, −1.671625279 + 0.9400455239 I]}

>
TimeSeries(*GeneNet*((0, 50, 0.2, 2, *m1*, *m2*, *m3*, *p1*, *p2*, *p3*), [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [0.2, 0.1, 0.3, 0.1, 0.4, 0.5], 0.1, 100, 1), [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [0, 0, 0, 0, 0, 0], 0.1, 100, 1)



18
16
14
12
10
8
6
4
2
0
10
20
30
40
50
60
70
80
90
100

>

>

>

>

10)

> $M := Matrix([[0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1], [0.1, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1], [0.1, 0.1, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1], [0.075, 0.075, 0.075, 0.4, 0.075, 0.075, 0.075, 0.075, 0.075], [0.075, 0.075, 0.075, 0.075, 0.4, 0.075, 0.075, 0.075, 0.075], [0.075, 0.075, 0.075, 0.075, 0.075, 0.4, 0.075, 0.075, 0.075], [0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.6, 0.05, 0.05], [0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.6, 0.05], [0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.6]])$

$$M := \begin{bmatrix} 0.2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.075 & 0.075 & 0.075 & 0.4 & 0.075 & 0.075 & 0.075 & 0.075 & 0.075 \\ 0.075 & 0.075 & 0.075 & 0.075 & 0.4 & 0.075 & 0.075 & 0.075 & 0.075 \\ 0.075 & 0.075 & 0.075 & 0.075 & 0.075 & 0.4 & 0.075 & 0.075 & 0.075 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.6 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.6 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.6 \end{bmatrix} \quad (1)$$

> M^{1000}

[[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160], **(2)**

[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],

[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],

[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],

[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],

[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160, 0.153846153846160, 0.153846153846160],

[0.0769230769230801, 0.0769230769230801, 0.0769230769230801, 0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160,

0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801,
0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160,
0.153846153846160, 0.153846153846160],
[0.0769230769230801, 0.0769230769230801, 0.0769230769230801,
0.102564102564107, 0.102564102564107, 0.102564102564107, 0.153846153846160,
0.153846153846160, 0.153846153846160]]