Workshop 4

1. Suppose that C_1 and C_2 are two curves in the plane, given parametrically by

$$C_1\colon \begin{cases} x(t) = 0 \\ y(t) = 2t - 1 \end{cases} \text{ for } 0 \le t \le 1\,; \quad C_2\colon \begin{cases} x(t) = -\cos t \\ y(t) = \sin t \end{cases} \text{ for } -\frac{\pi}{2} \le t \le \frac{\pi}{2}\,.$$

a) Sketch these curves.

b) If f is a function defined in the plane, let $I_1(f)$ and $I_2(f)$ be the line integrals of f over these curves with respect to arc length: $I_1(f) = \int_{C_1} f(x,y)\,ds$ and $I_2(f) = \int_{C_2} f(x,y)\,ds$. In each case below determine which of $I_1(f)$ and $I_2(f)$ is greater or whether they are equal (that is, whether $I_1(f) > I_2(f)$, $I_1(f) < I_2(f)$ or $I_1(f) = I_2(f)$) *without evaluating the integrals.* Explain your reasoning carefully. Then check your answer by computing the integrals.

i) $f(x,y) = 17$; ii) $f(x,y) = x$; iii) $f(x,y) = y$.

2. a) Create a two-dimensional force field $\mathbf{F} = M\mathbf{i} + N\mathbf{j}$ defined on all of $\mathbf{R}^2$ *except* $(0,0)$ with the following properties:

i) $\mathbf{F}$ is *always* perpendicular to the level curves of the function $g(x,y) = x^2 + 4y^2$.

ii) The magnitude of $\mathbf{F}$ at (x,y) is inversely proportional to the distance of (x,y) to the origin.

iii) $\mathbf{F}$ at $(1,0)$ is $\mathbf{i}$.

b) Compute $\int_C M\,dx + N\,dy$ where C is the curve given $\begin{cases} x = 2\cos(t^{78}) \\ y = \sin(t^{78}) \end{cases}$ $.34 \le t \le .56.$ (Think physically!)