

Workshop 3

1. Some of these iterated integrals correspond to real geometric problems (computation of volumes) and some do not. Please indicate which integrals are valid and which are not. Explain your answers. Compute any iterated integral which corresponds to a volume, and make a rough sketch of each volume computed.

$$a) \int_0^{x^3} \int_0^y x^2 + y^2 + 1 \, dx \, dy,$$

$$b) \int_0^1 \int_0^{5x} x^2 + y^2 + 1 \, dx \, dy,$$

$$c) \int_5^7 \int_{y^3}^{3y} x^2 + y^2 + 1 \, dx \, dy,$$

$$d) \int_{-1}^0 \int_{2y}^{-y^2} x^2 + y^2 + 1 \, dx \, dy$$

2. Suppose that $f(x, y) = (y - x^2)e^{x^2+y^2}$. Determine which of the integrals I_1 , I_2 , I_3 , and I_4 below is largest, which next largest, etc. Explain carefully how you reached your conclusions. Do not attempt to evaluate the integrals explicitly.

$$I_1 = \int_0^2 \int_0^4 f(x, y) \, dy \, dx$$

$$I_2 = \int_0^2 \int_0^{x^2} f(x, y) \, dy \, dx$$

$$I_3 = \int_0^2 \int_{x^2}^4 f(x, y) \, dy \, dx$$

$$I_4 = \int_0^2 \int_0^4 |f(x, y)| \, dy \, dx$$

Hint Where is f positive? Where negative?

3. Compute:

$$a) \int_0^{\frac{\pi}{2}} \left(\int_y^{\frac{\pi}{2}} \frac{\sin x}{x} \, dx \right) dy$$

$$b) \int_0^1 \left(\int_{\sqrt{y}}^1 e^{(7x^3)} \, dx \right) dy$$

$$c) \int_0^1 \left(\int_x^{x^{1/3}} \sqrt{1 - y^4} \, dy \right) dx$$

4. Suppose that $f(x, y)$ is some function of two variables and we wish to evaluate $I = \iint_R f(x, y) \, dA$, where R is the region in the xy -plane lying inside the circle $x^2 + y^2 = 4$ but outside the circle $(x - 1)^2 + y^2 = 1$. Set up (but do not try to evaluate) I as a sum of iterated integrals.