

Quiz 9
Math 250

Let A be a 2×2 matrix having eigenvalues 5 and -4 , and corresponding eigenvectors $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

- (1) What is the characteristic polynomial of A ?
- (2) Find A^3 .

- (1) $\det(A - tI_2) = (5 - t)(-4 - t)$
- (2) $A = PDP^{-1}$, where

$$P = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$$

and

$$D = \begin{bmatrix} 5 & 0 \\ 0 & -4 \end{bmatrix}.$$

We can also find the inverse of P , which is given by

$$P^{-1} = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}.$$

Now we know that

$$\begin{aligned} A^3 &= PD^3P^{-1} \\ &= \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 5^3 & 0 \\ 0 & (-4)^3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -(5)^3 & (-4)^3 \\ 3(5)^3 & -2(-4)^3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -2(5)^3 + 3(-4)^3 & -(5)^3 + (-4)^3 \\ 6(5)^3 - 6(-4)^3 & 3(5)^3 - 2(-4)^3 \end{bmatrix}. \end{aligned}$$