Quiz 8
Math 250

Let A be a 3×3 matrix such that $\det A = 3$. Evaluate the determinant of each matrix given below.

(1) A^3
(2) $(2A^T)^{-1}$
(3) $\begin{bmatrix} 4a_{21} & 4a_{22} & 4a_{23} \\ a_{11} + 5a_{31} & a_{12} + 5a_{32} & a_{13} + 5a_{33} \\ a_{31} - 2a_{21} & a_{32} - 2a_{22} & a_{33} - 2a_{23} \end{bmatrix}$

(1) $\det(A^3) = (\det(A))^3 = 3^3 = 27$

(2) $\det((2A^T)^{-1}) = \dfrac{1}{\det(2A^T)} = \dfrac{1}{2^3 \det(A^T)} = \dfrac{1}{8 \det(A)} = \dfrac{1}{(8)(3)} = \dfrac{1}{24}$

(3) The given matrix is obtained from A using the following elementary row operations:

$$\begin{aligned} 5r_3 + r_1 &\to r_1 \\ -2r_2 + r_3 &\to r_3 \\ r_1 &\leftrightarrow r_2 \\ 4r_2 &\to r_2 \end{aligned}$$

The first two row operations do not affect the determinant, but the third one changes it by a factor of -1 and the last one changes it by a factor of 4.
Therefore, $\det\left(\begin{bmatrix} 4a_{21} & 4a_{22} & 4a_{23} \\ a_{11} + 5a_{31} & a_{12} + 5a_{32} & a_{13} + 5a_{33} \\ a_{31} - 2a_{21} & a_{32} - 2a_{22} & a_{33} - 2a_{23} \end{bmatrix}\right) = (-1)(4)(3) = -12.$