

Quiz 2
Math 250

Consider the following system of linear equations:

$$\begin{aligned}x - 2y - z &= -3 \\ 2x - 4y + 2z &= 2.\end{aligned}$$

1. Write down the augmented matrix corresponding to this system of equations.
2. Find the reduced row echelon form of this augmented matrix.
3. Find the general solution of this system of equations if it is consistent, or explain why it is inconsistent.

1. $[A \mid \mathbf{b}] = \left[\begin{array}{ccc|c} 1 & -2 & -1 & -3 \\ 2 & -4 & 2 & 2 \end{array} \right]$

2.

$$\begin{aligned}\left[\begin{array}{ccc|c} 1 & -2 & -1 & -3 \\ 2 & -4 & 2 & 2 \end{array} \right] &\xrightarrow{-2r_1+r_2 \rightarrow r_2} \left[\begin{array}{ccc|c} 1 & -2 & -1 & -3 \\ 0 & 0 & 4 & 8 \end{array} \right] \\ &\xrightarrow{r_2/4 \rightarrow r_2} \left[\begin{array}{ccc|c} 1 & -2 & -1 & -3 \\ 0 & 0 & 1 & 2 \end{array} \right] \\ &\xrightarrow{r_2+r_1 \rightarrow r_1} \left[\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right]\end{aligned}$$

3. From the reduced row echelon form obtained in (2), we obtain the following:

$$\begin{aligned}x &= -1 + 2y \\ y &= \text{free} \\ z &= 2,\end{aligned}$$

so the general solution is given by:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} + y \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}.$$