

Quiz 1
Math 250

1. Let $\mathbf{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. Write down the rotation matrix corresponding to a clockwise rotation of 45° , and find the vector obtained by rotating \mathbf{u} in this way. (Note that $\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$.)

The rotation matrix corresponding to an angle of 45° *clockwise* is given by:

$$A_{-45} = \begin{bmatrix} \cos 45 & \sin 45 \\ -\sin 45 & \cos 45 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}.$$

To rotate \mathbf{u} in this way, we compute the matrix-vector product $A_{-45}\mathbf{u}$. This is given by:

$$\begin{aligned} A_{-45}\mathbf{u} &= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} \\ &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} \\ &= \frac{1}{\sqrt{2}} \left(3 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) \\ &= \frac{1}{\sqrt{2}} \begin{bmatrix} 4 \\ -2 \end{bmatrix} \\ &= \begin{bmatrix} 2\sqrt{2} \\ -\sqrt{2} \end{bmatrix}. \end{aligned}$$