

Quiz 10
Math 250

Find the eigenvalues and corresponding eigenvectors of the matrices given below.

$$(1) A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$(2) B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$(1) \det(A - tI_2) = \begin{vmatrix} (1-t) & 1 \\ 1 & (1-t) \end{vmatrix} = (1-t)^2 - 1 = t^2 - 2t = t(t-2) = 0$$

Eigenvalues = 0, 2

When $\lambda = 0$:

$$A - 0I_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \xrightarrow{-r_1 + r_2 \rightarrow r_2} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}.$$

Therefore:

$$x_2 : \text{free}$$

$$x_1 = -x_2$$

$$\text{Eigenvector} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

When $\lambda = 2$:

$$A - 2I_2 = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \xrightarrow{r_1 + r_2 \rightarrow r_2} \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}.$$

Therefore:

$$x_2 : \text{free}$$

$$x_1 = x_2$$

$$\text{Eigenvector} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$(2) \det(B - tI_2) = \begin{vmatrix} (1-t) & 1 \\ 0 & (1-t) \end{vmatrix} = (1-t)^2 = 0$$

Eigenvalues = 1, 1

When $\lambda = 1$:

$$B - 1I_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Therefore:

$$x_1 : \text{free}$$

$$x_2 = 0$$

$$\text{Eigenvector} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$