

## Math 152, Spring 2011, Formula Sheet for Exam 2

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$$\begin{aligned} \sin(0) = 0 ; \quad \sin(\pi/6) = 1/2 ; \quad \sin(\pi/4) = \sqrt{2}/2 ; \quad \sin(\pi/3) = \sqrt{3}/2 ; \quad \sin(\pi/2) = 1 \\ \cos(0) = 1 ; \quad \cos(\pi/6) = \sqrt{3}/2 ; \quad \cos(\pi/4) = \sqrt{2}/2 ; \quad \cos(\pi/3) = 1/2 ; \quad \cos(\pi/2) = 0 \end{aligned}$$

$$\cos^2 x + \sin^2 x = 1 ; \quad 1 + \tan^2 x = \sec^2 x ; \quad 1 + \cot^2 x = \csc^2 x$$

$$\sin(2x) = 2 \sin x \cos x ; \quad \cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos^2 x = \frac{1}{2}(1 + \cos(2x)) ; \quad \sin^2 x = \frac{1}{2}(1 - \cos(2x))$$

$$\sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$$

$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C ; \quad \int \csc x \, dx = \ln |\csc x - \cot x| + C$$

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If  $T_N$ ,  $M_N$ ,  $S_N$  are the Trapezoidal, Midpoint and Simpson's approximations, then

$$T_N = \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{N-2}) + 2f(x_{N-1}) + f(x_N)] ;$$

$$M_N = \Delta x [f(c_1) + f(c_2) + \cdots + f(c_N)] \text{ where } c_j = (x_{j-1} + x_j)/2 ;$$

$$S_N = \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{N-2}) + 4f(x_{N-1}) + f(x_N)] .$$

If  $I = \int_a^b f(x) \, dx$  then

$$|T_N - I| \leq \frac{K_2(b-a)^3}{12N^2} , \quad |M_N - I| \leq \frac{K_2(b-a)^3}{24N^2} , \quad |S_N - I| \leq \frac{K_4(b-a)^5}{180N^4} .$$

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The length of the curve  $y = f(x)$ ,  $a \leq x \leq b$  is equal to  $\int_a^b \sqrt{1 + (f'(x))^2} \, dx$ .

The area of the surface obtained by rotating the curve  $y = f(x)$ ,  $a \leq x \leq b$  about the  $x$ -axis is equal to  $\int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2} \, dx$ .

The length of the parametric curve  $(x(t), y(t))$ ,  $a \leq t \leq b$  equals  $\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} \, dt$ .

If a curve is given in polar form by  $r = f(\theta)$  then the area bounded by  $r = f(\theta)$ ,  $\theta = \alpha$  and  $\theta = \beta$  is  $\int_\alpha^\beta \frac{1}{2} r^2 \, d\theta = \int_\alpha^\beta \frac{1}{2} (f(\theta))^2 \, d\theta$ . The length of the polar curve  $r = f(\theta)$  between

$\theta = \alpha$  and  $\theta = \beta$  is  $\int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta = \int_\alpha^\beta \sqrt{(f(\theta))^2 + (f'(\theta))^2} \, d\theta$ .

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The  $n$ th Taylor polynomial of  $f(x)$  with center  $a$  is  $T_n(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x-a)^k$ .

If  $|f^{(n+1)}(u)| \leq K$  for all  $u$  between  $a$  and  $x$ , then  $|f(x) - T_n(x)| \leq K \frac{|x-a|^{n+1}}{(n+1)!}$ .