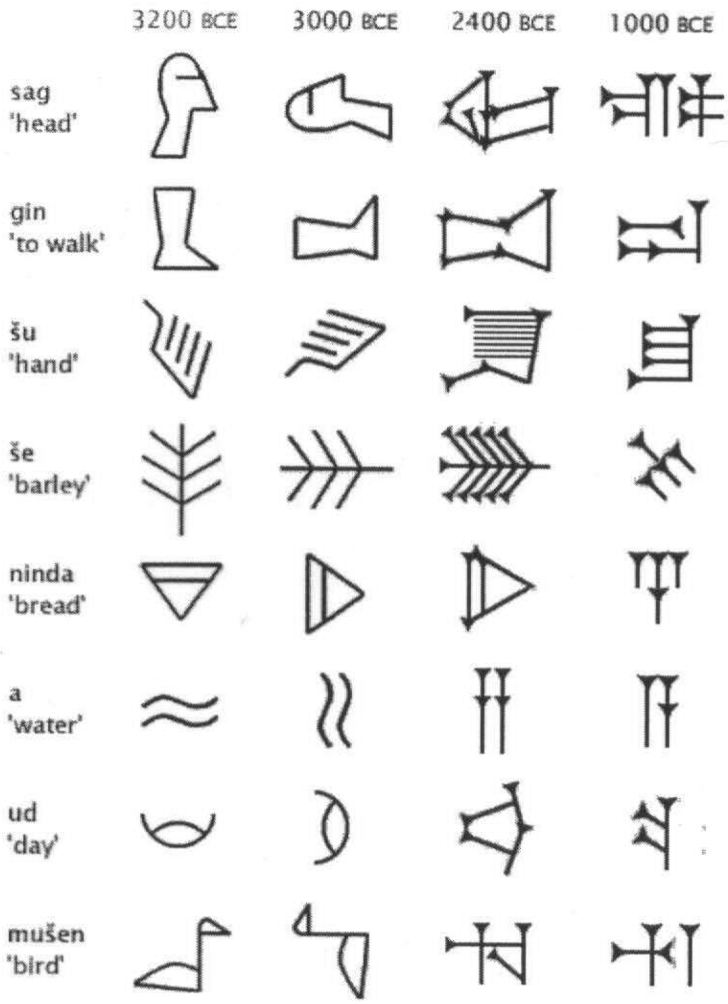
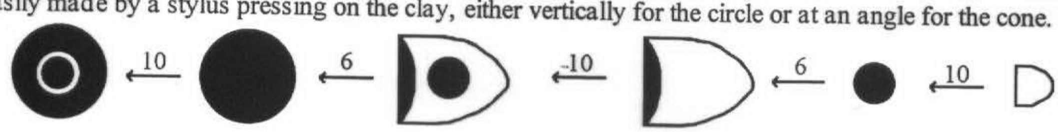



In the basic sexagesimal system used for counting most discrete objects, a single object, a sheep or cow or fish, is denoted by a small cone. Ten cones equaled one small circle, six small circles equaled one big cone, ten big cones equaled was a big cone with a circle inside it, six of those was a large circle and ten large circles was given by a large circle with a small circle inside. This last unit was then worth 10x6x10x6x10 = 36000 base units. Note that the circle and "cone-shape" could be easily made by a stylus pressing on the clay, either vertically for the circle or at an angle for the cone.

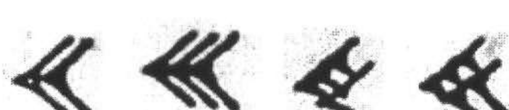


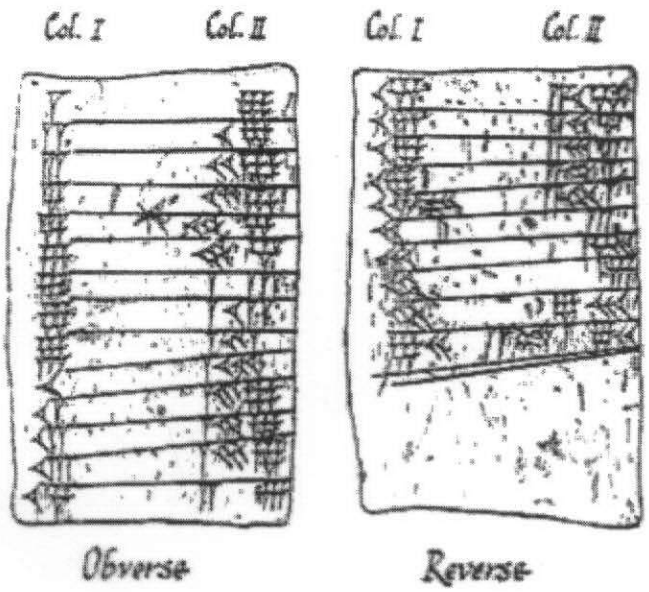

3000 BC

numbers 2 through 9 were written by combining multiples of a single stroke:

2000 BC

The number 10 was written in a single character  and the numbers 20 to 50 were written with multiples of this character:

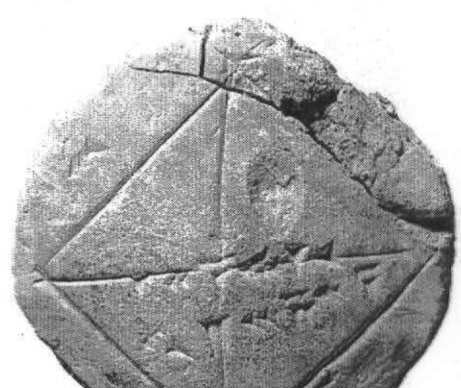




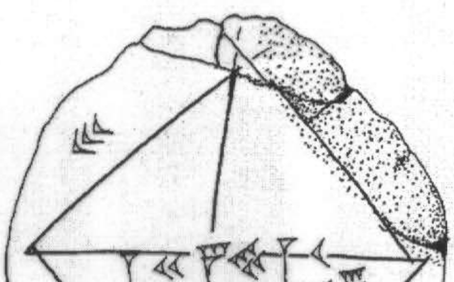

| | |
|---|---|
| 1 | 9 |
| 2 | 18 |
| ⋮ | |
| 9 | 81 = 1,21 |
| 10 | 90 = 1,30 |
| 11 | 99 = 1,39 |
| 12 | 108 = 1,48 |
| 13 | 117 = 1,57 |
| 14 | 126 = 2,6 |
| --- | |
| 20 | 180 = 3,0 |
| 30 | 270 = 4,30 |
| 40 | 360 = 6,0 |
| 50 | 450 = 7,30 |

# YBC 7289

The famous 'root(2)' tablet from the Yale Babylonian Collection.



Copyright: Yale Babylonian Collection



Copyright: A. Aaboe

$1,24 = 1.4 = 1 + \frac{24}{60}$

$1,24,51 = 1.41416$

$1,24,51,10 = 1.41421$

30 times this: 42.4264

or $42 + \frac{25}{60} + \frac{35}{60^2}$

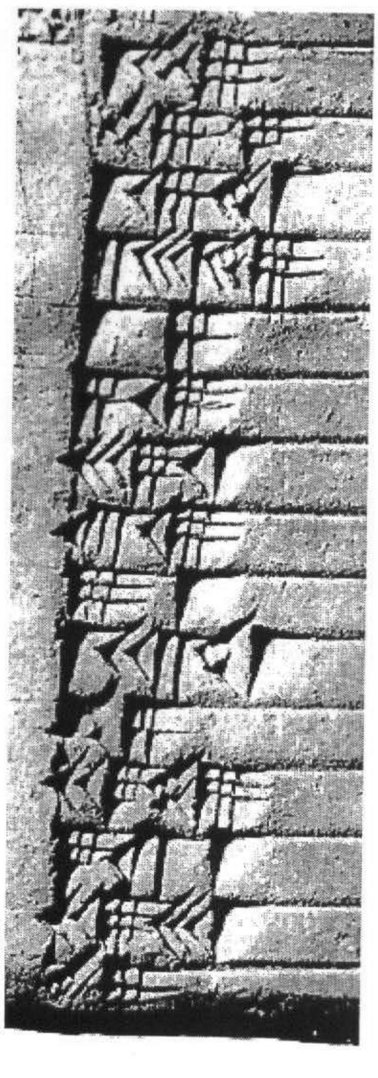
Notice that

$\frac{1}{\sqrt{2}} = 0.707106 = 0,42,25,35$

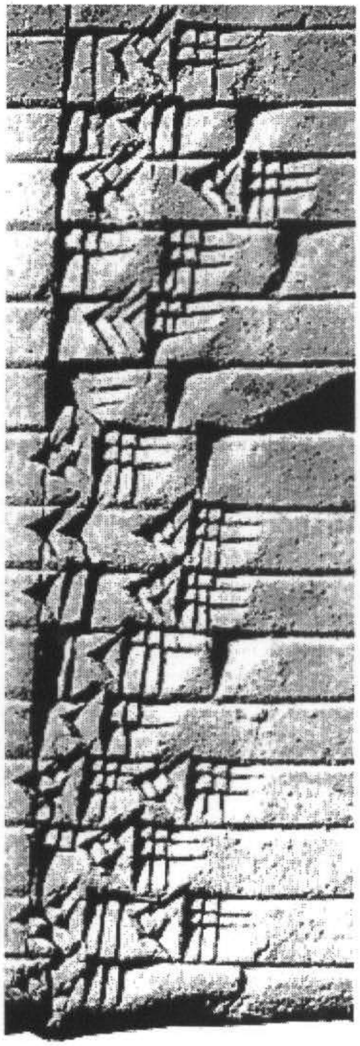
up to 6 decimal places

This tablet is a round school tablet of unknown provenance from the Old Babylonian period. It has a picture of a square with both diagonals drawn in. On one side of the square is written the number `30`, along one of the diagonals is the number `1,24,51,10` and below it is `42,25,35`.

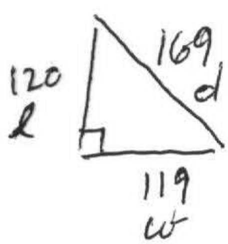
# The Babylonian tablet Plimpton 322

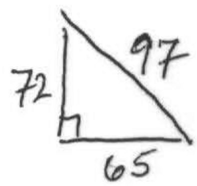


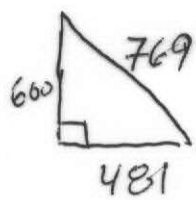
| "width" | | "kī" (number) |
|---|---|---|
| 1,59 | 119 | 1 |
| 56,7 | 3367 | 2 |
| 1,16,41 | 4601 | 3 |
| 3,31,49 | 12709 | 4 |
| 1,5 | 65 | 5 |
| 5,19 | 319 | 6 |
| 38,11 | 2291 | 7 |
| 13,19 | 799 | 8 |
| 9,1 [8,1] | 541[481] | 9 |
| 1,22,41 | 4961 | 10 |
| 45 | 45 | 11 |
| 27,59 | 1679 | 12 |
| 7,12, 1 [2,41] | 25921[161] | 13 |
| 29,31 | 1771 | 14 |
| 56 | 56 | 15 |

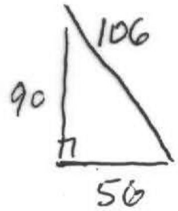


| "diagonal" | | |
|---|---|---|
| 2,49 | 169 | $169^2-119^2=120^2$ |
| 3,12,1 [1,20,25] | 11521 [4825] | |
| 1,50,49 | 6649 | |
| 5,9,1 | 18541 | |
| 1,37 | 97 | $97^2-65^2=72^2$ |
| 8,1 | 481 | |
| 59,1 | 3541 | |
| 20,49 | 1249 | |
| 12,49 | 769 | $769^2-\underline{481}^2=600^2$ |
| 2,16,1 | 8161 | |
| 1,15 | 75 | |
| 48,49 | 2929 | |
| 4,49 | 289 | |
| 53,49 | 3229 | |
| 53 [1,46] | 53 [106] | $\underline{106}^2-56^2=90^2$ |




$\frac{d}{l}=\frac{169}{120}=1.408$

$\left(\frac{169}{120}\right)^2=1.9834\ldots$

$=1,59,0,15$

$\left(\frac{d}{l}\right)^2=\left(\frac{769}{600}\right)^2$

$=1.6427$

$=1,38,33,36,36$

$(d/l)^2=\left(\frac{106}{90}\right)^2$

$=1.38716$

$=1,23,13,46,40$

| | |
|---|---|
| 1,59,0,15 | 1.9834 ... |
| 1,56,56,58,14,50,6,15 | 1.94916 ... |
| 1,55,7,41,15,33,45 | 1.9188 ... |
| 1,53,10,29,32,52,16 | 1.88625 ... |
| 1,48,54,1,40 | 1.81501 ... |
| 1,47,6,41,40 | 1.78519 ... |
| 1,43,11,56,28,26,40 | 1.71998 ... |
| 1,41,33,59,3,45 | 1.6928 ... |
| 1,38,33,36,36 | 1.64267 ... |
| 1,35,10,2,28,27,24,26 | 1.58612 ... |
| 1,33,45 | 1.5625 ... |
| 1,29,21,54,2,15 | 1.48942 ... |
| 1,27,0,3,45 | 1.45002 ... |
| 1,25,48,51,35,6,40 | 1.43024 ... |
| 1,23,13,46,40 | 1.38716 ... |

## What it means

1945

Following perhaps the hints given by the words *diagonal* and *width*, Neugebauer and Sachs discovered that if $w$ is the entry in the second column and $d$ is the entry in the third column, then in all but a few cases $d^2$-$w^2$ turned out to be a perfect integer square $l^2$. In other words, assuming the exceptions to be caused by error, this table contains part of a list of Pythagorean triples, that is to say integers $w$, $l$, $d$ with

$$w^2 + l^2 = d^2$$

which form the sides of right triangles. Here is the resulting table of calculations, in modern notation (with discrepancies in square brackets):