Gabriel Cramer (1704-1752)

Born in Geneva, Switxerland

In 1728 he proposed a solution to the **St. Petersburg paradox** (based on a lottery in St. P). It was solved 1738 by Dan Bernoulli

"Introduction à l'analyse des lignes courbes algébraique", 1750.

It contains the earliest demonstration that a curve of degree n is determined by $n(n + 3)/2$ points on it.

(He forgot that the points must be in general position).

Cramer's paradox: he states a theorem by Maclaurin which says that an equation of degree n intersects an equation of degree m in nm points. Taking $n = m = 3$ this says that two cubics intersect in 9 points, yet Cramer's formula $n^2/2 + 3n/2$ with $n = 3$ gives 9 so a cubic is uniquely determined by 9 points. This, says Cramer, is a paradox.

Euler finally resolved this paradox in 1750. (a) the points must be in general position to uniquely determine the curve, because the equations must be linearly independent.
(b) Given 8 points (no 4 collinear, no 7 on a conic), there is a unique 9[th] point on any cubic through 'em

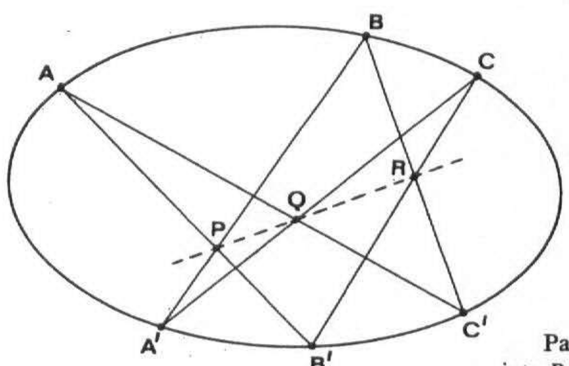


Figure 22. Pascal's theorem.

Pascal's theorem: if A,B,C,A',B',C' are any six points on a conic, then the points $P = AB'.A'B$, $Q = AC'.A'C$, and $R = BC'.B'C$ are collinear (Fig. 22).

Alexandre-Théophile Vandermonde

(1735-1796)

He pursued a music career and he only turned to mathematics when he was 35 years old (1770). Vandermonde was a strong supporter of the Revolution (1789), left mathematics research after that.

Mémoire sur la résolution des équations (1771)

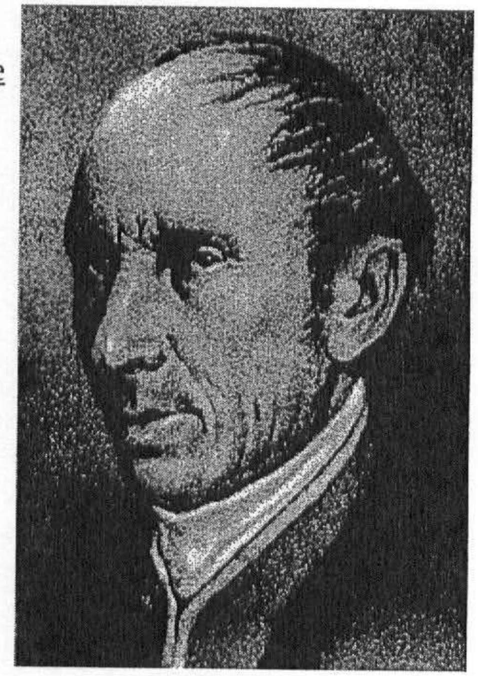
$$\left\{\begin{array}{l} a_{00} + a_{10}x + a_{20}x^2 \\ +a_{01}y + a_{11}xy \\ +a_{02}y^2 \end{array}\right\}$$

6 unknowns, each point (x,y) gives equation

A conic is determined by 5 points* (5 equations in 5 unknowns)
* no 3 on a line

Augustin Louis Cauchy (1789-1857)

Staunch royalist family, fled during French Revolution until 1796.
Cauchy was an Engineer 1807-1813, degree from l'École Nationale des Ponts et Chaussées
Worked on port facilities for Napolean's English invasion.
1813-1830 he was professor at the École Polytechnique,
but resigned rather than swear fealty to King Louis-Philippe
Cauchy's views were widely unpopular among mathematicians
Rest of career in Torino (Italy)



Cauchy coined the word determinant in an 1815 paper, as well as adjoint matrix, and det(A)det(B)=det(AB).
Numerous terms in mathematics bear Cauchy's name:

- the Cauchy integral theorem, in the theory of complex functions,
- the Cauchy-Kovalevskaya existence theorem for the solution of partial differential equations,
- the Cauchy-Riemann equations and Cauchy sequences.

The word matrix was coined in 1850 by Sylvester, but not used in print until 1855 (by Cayley)

Discovered the *characteristic polynomial* $\det(A-\lambda)=0$ and used it to find the characteristic values λ of A (now called *eigenvalues*)

Find max, min on unit circle of $f(x,y) = ax^2+2bxy+cy^2$. Eigenvectors of $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$

Charles Lutwidge Dodgson (1832-1898)

"Carolus Lodovicus" in Latin, anglicized as "Lewis Carroll"

A syllabus of plane algebraical geometry (1860)
Alice's adventures in wonderland (1865)
Condensation of Determinants (1866)
Elementary Treatise on Determinants (1867)

Conditions on an mxn matrix A and vector **b** so that A**x**=**b** is consistent.
(rank of a matrix)
If A has rank r and every set of r+1 equations has a solution, then there is a solution.
In fact there are *n-r* independent solutions.
His proof was constructive, using the augmented matrix (A|**b**)

"independence" due to Frobenius (1879)

Johann Carl Friedrich Gauss (1777-1855)

Born in Brunswick (Dutchy of Brunswick). At age 7 added up numbers 1-100.
During 1795-98, he studied at Göttingen University, but left without a degree.
Solved construction of regular 17-gon, and quadratic reciprocity.
In 1806, Duke of Brunswick killed in battle, and Gauss left for Göttingen.
In 1809, published *Disquisitiones Arithmeticae*. Containing many new results.

In 1801, he used lest squares to correctly predict where the "planet" Ceres would reappear.
To do this, he invented what we now call Gauss elimination to solve systems of linear equations.
Theoria motus corporum coelestium in sectionibus conicis Solem ambientium, in 1809

Gauss was asked in 1818 to carry out a geodesic survey of the state of Hanover. This led to the study of curvature, and Differential Geometry.
Theoria combinationis observationum erroribus minimis obnoxiae (1823), handbook of Statistics

In 1831, Wilhelm Weber arrived in Göttingen as physics professor, worked with Gauss on E-M
In six years, they discovered many things (Kirkhoff's Laws, 2 magnetic poles on Earth,...)

Marie-Sophie Germain (1776-1831)

Born to a middle-class family in Paris
Decided to become a mathematician in 1789 (!)
after reading about Archimedes' death by soldiers
Her father supported her financially all her life.

Used the pseudonym M. LeBlanc to submit an article
to Lagrange.

In 1804 - 1809 she wrote a dozen letters to Gauss, initially under the pseudonym "M. LeBlanc" because she feared being ignored because she was a woman. During their correspondence, and in letters to other mathematicians, Gauss gave her proofs high praise.

Germain's true identity was revealed to Gauss only after the 1806 French occupation of his hometown of Braunschweig. Recalling Archimedes' fate and fearing for Gauss's safety, she contacted a French commander who was a friend of her family.

1808 French Academy offered prize to *"formulate a mathematical theory of elastic surfaces"*
Only person to submit a solution 1811, 1813, 1815 but never appeared to receive the prize.

Sophie Germain prime is a prime number q where $q=2p+1$ and p is also prime. (e.g, $p=59$)

Fermat's Last Theorem: $a^n + b^n = c^n$ has no solutions in non-zero integers a, b, and c.

If true for n then true for mn because $a^{np} + b^{np} = c^{np}$ implies that $(a^p)^n + (b^p)^n = (c^p)^n$
Solved for n=4 by Fermat, so we can assume that n is prime: $a^p + b^p = c^p$

Case one: p does not divide abc. (Case two is when p divides one of a, b, c)
Euler solved FLT for n=3 in case one, and found a similar argument in case two.
For $x,y,x<10^{30}$: Sophie Germain proved there are no solutions to $x^5 + y^5 = z^5$.

In 1819, Germain proved Case one of FLT for all $p<100$.
The key: if $q=2p+1$ is a Germain prime then case two does not occur for p.

The proof when n=5 (in case 2) was proven in 1825 by Dirichlet and Legendre.
The case n=7 was found by Lame in 1839
Kummer proved FLT for all regular primes in 1847. (The first irregular primes are 37,59,67, 101, 103)