1

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ا | ب | ج | د | ه | و | ز | ح | ط |
| A | B | G | D | H | W | Z | Ḥ | Ṭ |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| ى | ك | ل | م | ن | س | ع | ف | ص |
| I | K | L | M | N | S | c | F | Ṣ |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| ق | ر | ش | ت | ث | خ | ذ | ض | ظ |
| Q | R | Sh | T | Th | Kh | Dh | Ḍ | Ẓ |
| 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| | | | | غ | | | | |
| | | | | Gh | | | | |
| | | | | 1000 | | | | |

Zero is ζ or ʊ

$\boxed{A \ KD}$ is 84
1 20+4

$\boxed{LB \ N}$ is
32 50/60
(Base 60)

Fig. 2.8

Base 60 needed for astronomy

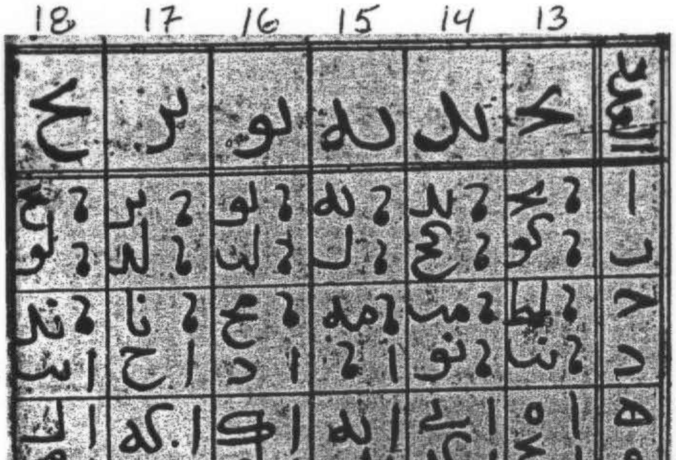



Plate 2.1. Part of a sexagesimal multiplication table. The right-most column is headed "the number" and shows the alphabetic numerals from 1 to 12. The succeeding columns (from right to left, as in Arabic handwriting) are headed by the numerals 13, 14, . . . , 18 and the entries underneath them give their multiples expressed as two-place sexagesimals. (See Fig. 2.12 for a transliteration and translation of the right-most three columns of this table.) (Photo courtesy of the Egyptian National Library.)

2



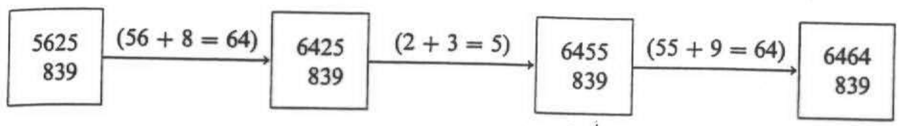



Fig. 2.1 (addition)

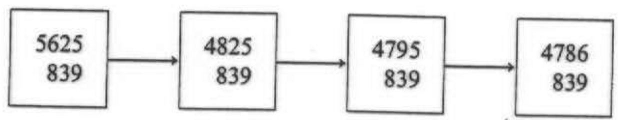



Fig. 2.2 (subtraction)

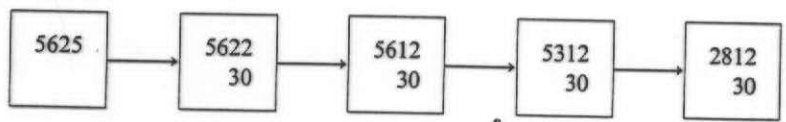



Fig. 2.3 (halving)

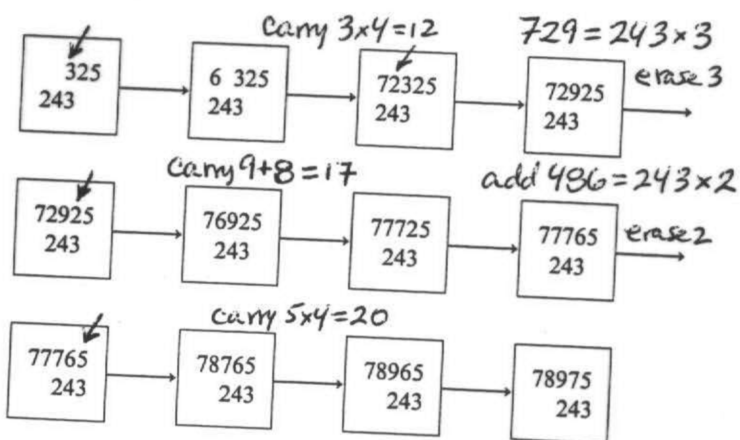
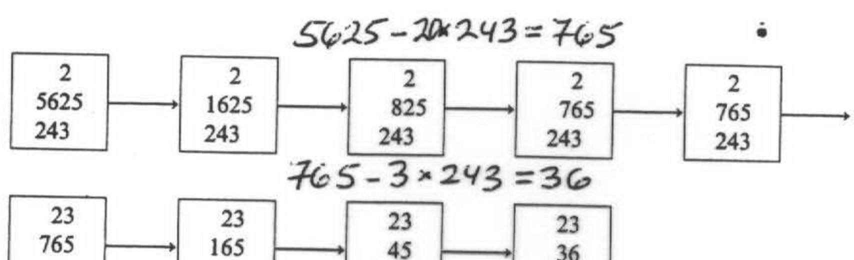
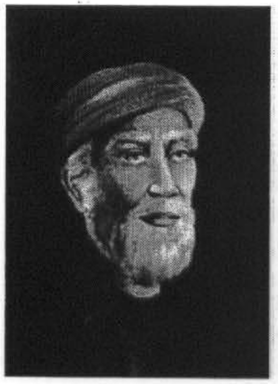



Fig. 2.4 multiply 243×325




Dividing Fig. 2.5 $\frac{5625}{243} = 23 + \frac{36}{243}$

813-833 House of Wisdom founded Baghdad



## Al Biruni (973-1055 AD)
## (Abu Raihan Mohamad ibn Ahmed Al Biruni)

This great historian mathematician astronomer, philosopher geologist and mineralogist was born in one of the suburds of Khwarizm. One of his famous works was his book Kitab Ul Hind which he wrote during his stay in India and Qumud Al Masud (an astronomical encyclopedia) On the mathematical and astronomical side of Geography he discussed the antipodes and roundity of the earth, the determination of its movement and gave the latitudes and longitudes of numerous places.

## Al Khwarizmi (780-850 AD)
## (Mohamad ibn Musa Al Khwarizmi - born in Khwarizm (Uzbek)

This mathematician and astronomer's work is the first golden period of Islamic civilization. Born in the city of ~~Baghdad~~ Al Kwarizmi was the first original mathematician in the world. His great contribution came in his book Hisab Al Jabr wal Muqubalah which laid the foundation for the science of algebra. Al Khwarizmi was also the first great Muslim geographer who wrote the book, Surat Al Arz or the shape of the Earth. Together with other 69 scholars he gave a map of the world and this was one of the first map in the world.

## Umar Hayyam (1048-1131 AD)
## Umar Ibn Ibrahim Al Khayyam

Al Khayyam was a great Muslim mathematician, poet and astronomer. Some of his works include treatises on arithmetic, algebra and astronomy. His solution of the euluc and liquadratic with the help of conic section is the most advanced work in Mathematics so far.
(cubic equation)

## Al Battani (850-929 AD)
## Abu Abdallah Mohammad ibn Jabir Al Battani

Born 850 in Harran, Al Battani was a famous astrologer and leader in geometry and astronomy. His achievements include inventing formulae for right angled triangles such as b sin (A) = a sin (90-A), cataloguing 489 stars, detailing the existing values for the length of the year (365 days 5 hours 48 minutes 24 seconds, and the seasons), calculating 54.5" per year for the precession of the equinoxes and obtained the value of 23 35' for the inclination of the ecliptic. He also showed that the farthest distance of the Sun from the Earth varies and, which can explain the annular and total eclipses of the Sun.

## al-Khwarizmi
## (Muhammad ibn Musa al-Khwarizmi)

Arab mathematician, b. c. **780** (Khwarizm), d. c. **850**.

---

Several works of Al-Khwarizmi are known. He wrote the *Mafatih al-'Ulum* ("Key to the Sciences"), the first Arabic encyclopaedia of knowledge that was organized on scientific principles.

His work on arithmetic was translated into Latin in the 12th century, and although the original is lost, the Latin translation *Algoritmi de numero Indorum* ("The al-Khwarizmi on the Indian numbers") still exists. Its title gave rise to the mathematical term "algorithm."

Al-Khwarizmis' other work, *Kitab al-jabr wa l-muqabala* ("The Book of Reduction and Comparison"), was a synthesis of Indian algebra and Greek geometry and had the most profound effect on the development of science. Latin translations, summaries and commentaries were written from the 12th century onward. The mathematical term "algebra" was derived from its title.

The *Kitab al-jabr wa l-muqabala* gives solutions to equations through the process of *al-jabr* and *al-muqabala:* An equation is first transformed into one to which the solution is known. Six such equations are given:

$$
\begin{aligned}
ax^2 &= bx \\
ax^2 &= c \\
bx &= c \\
ax^2 + bx &= c \\
ax^2 + c &= bx \\
bx + c &= ax
\end{aligned}
$$

$$x^2+bx=c$$
$$x^2+10x=39$$
$$x=\sqrt{\left(\frac{b}{2}\right)^2+c}-\frac{b}{2}$$

where all numbers are positive. The solutions to these equations are given as algorithms, and their proofs are given through geometrical argument. Babylonian-style geometric proof

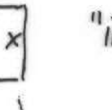
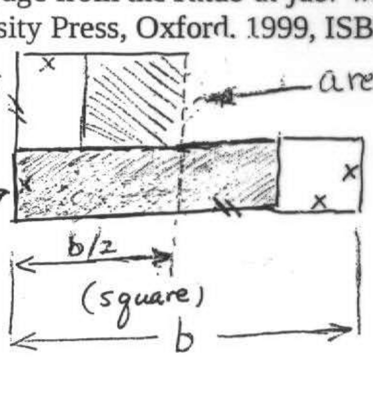



Illustration: a page from the *Kitab al-jabr wa l-muqabala,* J. L. Espositio (ed): Oxford History of Islam, Oxford University Press, Oxford. 1999, ISBN 0195107993; public domain *(Wikipedia)*

$x^2+c=bx$

area c →



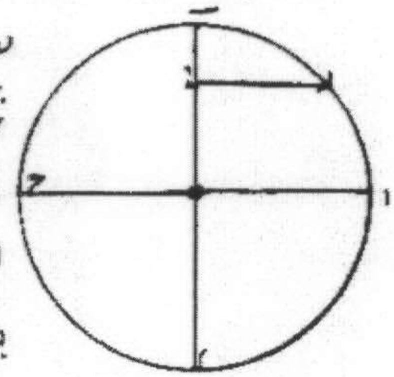

area $(b/2)^2-c=\Delta$

"impossible" if $\Delta<0$

هذه رسالة لأبي الفتح عمر بن ابراهيم الخيامي بسم الله الرحمن الرحيم رب يسر وتمم

نريد أن نقسم ربع دائرة [illegible] على نقطة مثل [illegible] ونخرج عمود [illegible]

على قطر [illegible] فيكون نسبة آه الى [illegible] كنسبة [illegible] مركز الدائرة وآه نصف القطر



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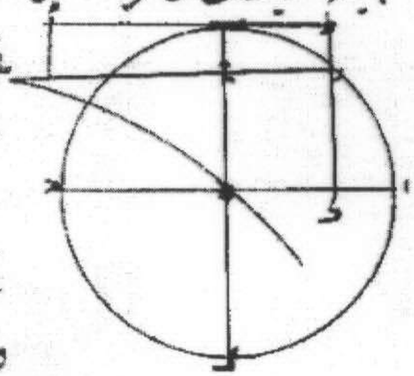

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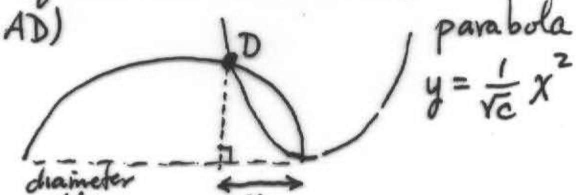
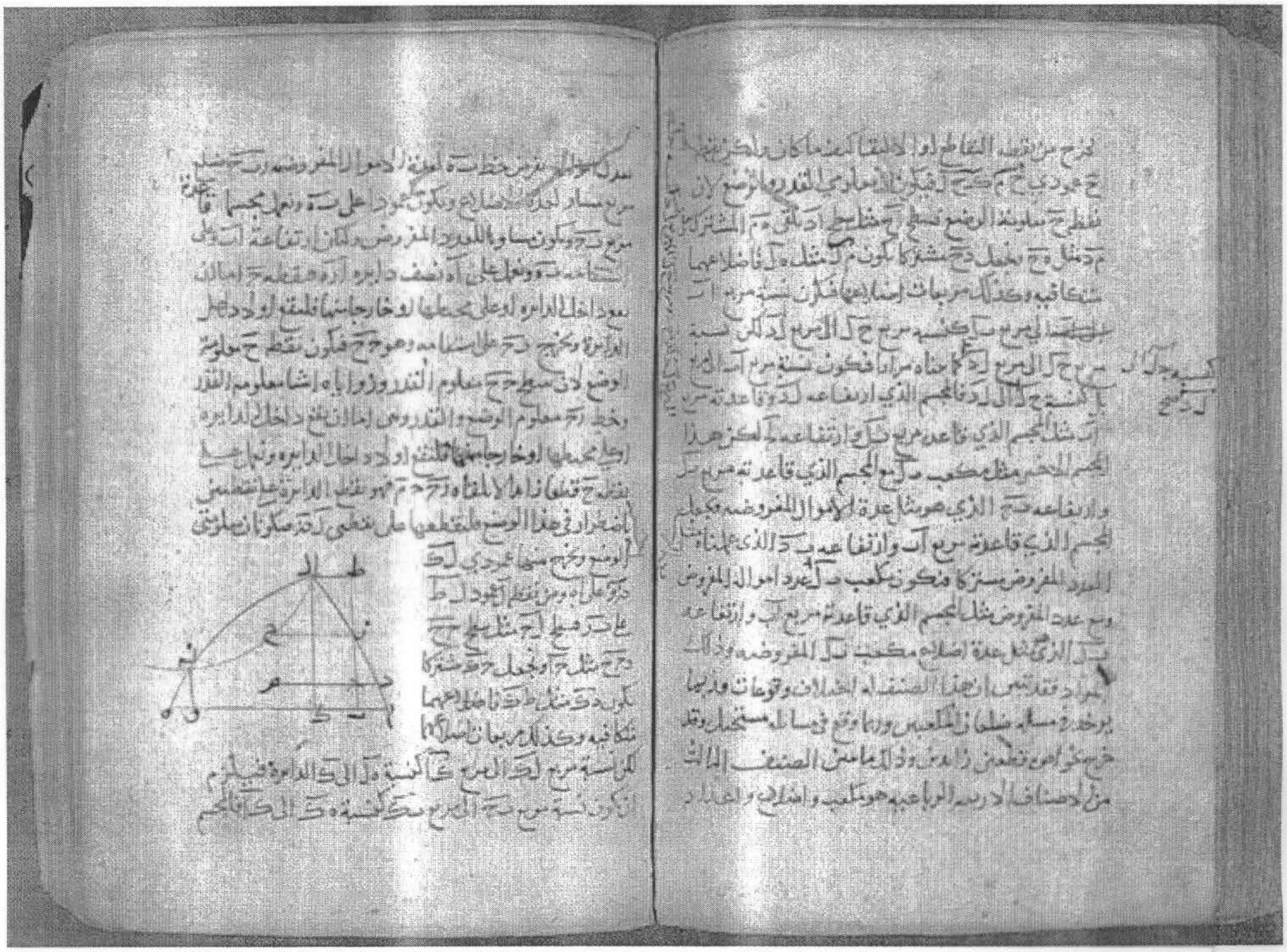
First page of "Cubic Equation and intersection of Conic sections"

Omar Khayyam (1080 AD)

$x^3 + cx = d$





**Omar Khayyam (1048-1122 CE). Maqalah fi al-jabr wa-al muqabalah.**

Manuscript on paper, 56 leaves. Lahore, India, 13th century.

Although the "Algebra" was unknown to western mathematicians until the eighteenth century, Omar received wide recognition for it in the Islamic world. Among the other fourteen works bound in this volume are two by Sharaf al-Din al Tusi (d. ca. 1213/1214)

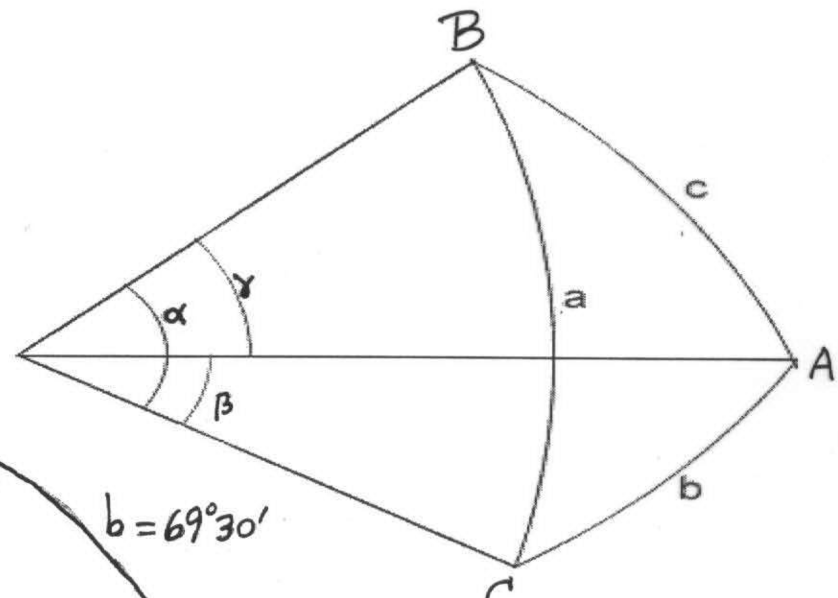
Spherical trigonometry is one method with which to find the distance between two points on a sphere.

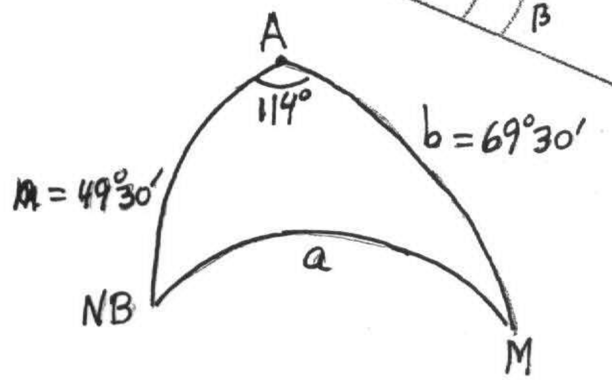
- Ptolomy (150 AD) studied these.
  - The sides of spherical triangles are segments of great circles.
  - The sum of the internal angles of a spherical triangle is not always 180°.
  - The law of cosines for a spherical triangle is:

$a = R\alpha,\ b = R\beta,\ c = R\gamma$

$$\boxed{\cos(a) = \cos(b)*\cos(c) + \sin(b)*\sin(c)*\cos(A)}$$

- The law of sines states that $\boxed{\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}}$




$$\cos(a) = \cos(m)\cos(b) + \sin(m)\sin(b)\cos(A)$$
$$= (.64)(.35) + (.76)(.93)(-.40)$$
$$= -0.0605$$
$$a = 93.47° \quad (93°29') \quad (1.63 \text{ radians})$$

This is 6465 miles as R = 3963 miles

| Where? | Longitude | Latitude |
|---|---|---|
| New Brunswick | 40 degrees 30' E | 74 degrees W |
| Mecca | 21 degrees 29' E | 40 degrees E |
| London | 0 degrees 4' W | 51 degrees 34' N |

$\dfrac{a}{\sin(A)} = \dfrac{b}{\sin(NB)}$ yields $\sin(NB) = \left(\dfrac{69}{93}\right)(0.9135) = 0.679$

so $\boxed{NB = 42°47'}$ (northeast direction) to Mecca

The local gibla

# Linear Interpolation

al-Biruni
1000 AD

| درج | دقائق | الجيوب | | | | التعاديل | | | | الفصول | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| سطر | عدد القسي | دقائق | ثواني | ثوالث | روابع | دقائق | ثواني | ثوالث | روابع | ثواني | ثوالث | روابع |
| ٠ 0 | يه 15 | ٠ | يه | مب | كح | ا | ب | مط | نب | يه | مب | كح |
| ٠ 0 | ل 30 | ٠ | لا | كد | نو | ا | ب | مط | م | يه | مب | كه |
| ٠ 0 | مه 45 | ٠ | مز | ز | كا | ا | ب | مط | كح | يه | مب | كب |
| ا 1 | ٠ 0 | ا | ب | مط | مج | ا | ب | مط | يب | يه | مب | يج |
| ا | يه | ا | يح | لب | ا | ا | ب | مح | مح | يه | مب | يب |
| ا | ل | ا | لد | يد | يج | ا | ب | مح | كد | يه | مب | و |
| ا | مه | ا | مط | نو | يط | ا | ب | مز | نب | يه | ما | غ |

Fig. 5.17(a)

| Degrees | Minutes | Sines | | | | Differences | | | Corrections | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Column of Arc number | | Minutes | Seconds | Thirds | Fourths | Seconds | Thirds | Fourths | Minutes | Seconds | Thirds | Fourths |
| 0 | 15 | 0 | 15 | 42 | 28 | 15 | 42 | 28 | 1 | 2 | 49 | 52 |
| 0 | 30 | 0 | 31 | 24 | 56 | 15 | 42 | 25 | 1 | 2 | 49 | 4 |
| 0 | 45 | 0 | 47 | 7 | 21 | 15 | 42 | 22 | 1 | 2 | 49 | 2 |
| 1 | 0 | 1 | 2 | 49 | 43 | 15 | 42 | 18 | 1 | 2 | 49 | 1 |
| 1 | 15 | 1 | 18 | 32 | 1 | 15 | 42 | 12 | 1 | 2 | 48 | 48 |
| 1 | 30 | 1 | 34 | 14 | 13 | 15 | 42 | 6 | 1 | 2 | 48 | 2 |
| 1 | 45 | 1 | 49 | 56 | 19 | 15 | 41 | 58 | 1 | 2 | 47 | 5 |

carry

$$\begin{array}{r} 0;0,31,24,56 \\ -\ 0;0,15,42,28 \\ \hline 0;0,15,42,28 \\ \times 4 \\ \hline 0;1,2,49,52 \end{array}$$

carry 2 carry 1

Fig. 5.17(b). Transcription of part of al-Bīrūnī's Sine Tables

$$\sin\left(\tfrac{1}{4}^\circ\right) = 0.0043633$$
$$= \frac{1}{3600}(15.7079) = \frac{15}{3600} + \frac{42.4748}{60^3}$$
$$= \frac{15}{60^2} + \frac{42}{60^3} + \frac{28}{60^4} + \frac{29}{60^5} + \frac{18}{60^6}$$
$$= (0;0,15,42,28,29,18)$$

$60\times(0.7079) = 42.4748$

$60\times(0.4748) = 28.48833$

$60\times(0.48833) = 29.2998$

Example: $\sin(1^\circ 22') = \sin(1^\circ 15') + \underset{\Delta\theta}{(0;7)}\underbrace{(0;1,2,48,48)}_{4\,\Delta \sin}$
$$= (0;1,18,32,1) + (0;0,7,19,41)$$
$$= (0;1,25,51,42)$$

$$\begin{array}{r} 5\ 5 \\ 0;1,2,48,48 \\ 7 \\ \hline 0;7,19,41,36 \end{array}$$

Calculator
$0.023850 = (0;1,25,51,43,24,14)$

$60^6 = 5\times 10^{10}$
(limit of accuracy)

# Non-linear interpolation

Giyath alDin al-Kashi (Samarkand, 1400+AD)

Triple angle formula $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$

$$\boxed{c = 3y - 4y^3}$$

iterative scheme: $y_{n+1} = \frac{c}{3} + \frac{4}{3} y_n^3$

Example al-Kashi knew $c = \sin(3°) = 3;8,24,33,59,\ldots$ , $g = 47,6;8,29,53,37,3,45$

$p = 45,0 = 2700$

$c/3 = 0.0174453 = \frac{g}{p}$

Modern: Start with $y_1 = 0.02$

$y_2 = (0.0174453)^{187} + 0.$
$+0.000\,10666$
$= 0.0174\cancel{5}6519$
etc

al-Kashi: Replace $y$ by $60y$, $\frac{c}{3}$ by $\frac{60c}{3} = \frac{g}{p}$ $\boxed{y = \frac{g}{p} + \frac{y^3}{p}}$

write $y = 1 + b + c + \cdots$ where $b$ is order $1/60$
$c$ is order $1/60^2$
$\ldots$

$(y_1 = g/p)$ $\quad y_2 = 1 + b = \frac{g + a^3}{p}$ gives $b = 2/60$

$y^3 = 1 + b + c = \frac{g + (1+b)^3}{p}$ gives $c = 49/60^3$

$y^4 = \frac{g + y^3}{p}$ gives $d = 43/60^4$ or $y \cong (1;2,49,43)$

worked it out up to $\frac{17}{60^9}$ term $(\cong 1 \times 10^{-15})$ beyond hand calc!

# Al-Samaw'al
## 1172 AD Baghdad

- introduced negative coefficients, polynomials
- law of exponents (table of powers)

$$\cdots\ 7\ \cdots\ 3\quad 2\quad 1\quad 0\quad 1\quad 2\quad 3$$
$$\cdots\ x^7\ \cdots\ x^3\quad x^2\quad x\quad 1\quad 1/x\quad 1/x^2\quad 1/x^3$$

"to multiply $x^3 \cdot x^4$, count 4 orders to the left of $x^3$. To divide count 4 orders to right."

- first to see that we can approximate fractions as a sequence of answers, "each more precise"

$\frac{210}{13}$ is 16.15384 "does not come out evenly"
16.153846
16.$\underline{153846}$153846…

* modern notation for decimal fractions – 1400 AD al-Kashi

- Divide polynomials

$$\begin{array}{r l}
 & 3\tfrac{1}{3} + 5/x - 6\tfrac{2}{3}(1/x^2) - 10(1/x^3) + 13\tfrac{1}{3}(1/x^4) + 20(1/x^5) \\
6x^2+12 \,\big) & 20x^2 + 30x \\
 & \underline{20x^2 + 0 + 40} \\
 & 30x - 40 \\
 & \underline{30x + 0 + 60(1/x)} \\
 & -40 - 60(1/x) \\
 & \underline{-40 - 0 - 80(1/x^2)} \\
 & -60(1/x) + 80/x^2 \\
 & \underline{-60(1/x) + 0 - 120(1/x^3)} \\
 & 80(1/x^2) + 120(1/x^3)
\end{array}$$

$(-2)(6\tfrac{2}{3})$ $(-2)(10)$

see the pattern?

Math 351:

answer is

$$\boxed{\begin{array}{l}(20x^2+30x) = (3\tfrac{1}{3})(6x^2+12) + R \\ \text{Remainder } R \text{ is } 30x - 40\end{array}}$$

"He then proudly wrote out the terms out to $54613\frac{1}{3}\left(\frac{1}{x^{28}}\right)$"

<u>Modern</u>: $\frac{1}{6x^2+12} = \frac{1/12}{1+\frac{1}{2}x^2} = \frac{1}{12}\left(1 - \frac{1}{2}x^2 + \frac{1}{4}x^4 - \frac{1}{8}x^6 + \frac{1}{16}x^8 \cdots\right)$

multiply by $20x^2+30x = 12\left(3\tfrac{1}{3}x^2 + \tfrac{5}{2}x\right)$ to get power series!