Sulba Sutras

The most important of these documents are the

Baudhayana Sulbasutra written about 800 BC;

Manava Sulbasutra written about 750 BC;

Apastamba Sulbasutra written about 600 BC.;

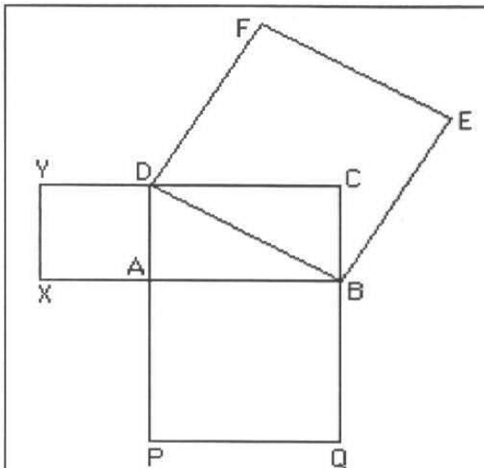
Katyayana Sulbasutra written about 200 BC.

Pythagoras's theorem.

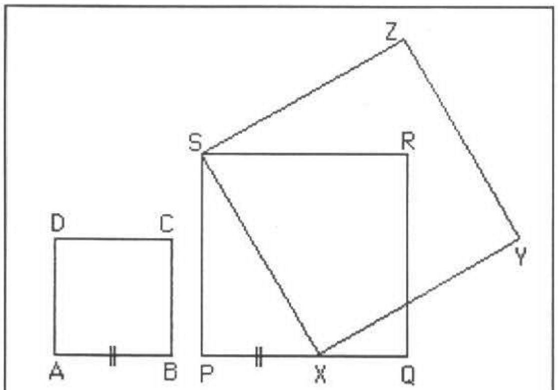
There are many examples of Pythagorean triples in the Sulbasutras. For example (5, 12, 13), (12, 16, 20), (8, 15, 17), (15, 20, 25), (12, 35, 37), (15, 36, 39), $({}^{5}/_{2}, 6, {}^{13}/_{2})$, and $({}^{15}/_{2}, 10, {}^{25}/_{2})$ all occur.

Baudhayana Sulbasutra : *The rope which is stretched across the diagonal of a square produces an area double the size of the original square.*

Katyayana Sulbasutra: *The rope which is stretched along the length of the diagonal of a rectangle produces an area which the vertical and horizontal sides make together.*

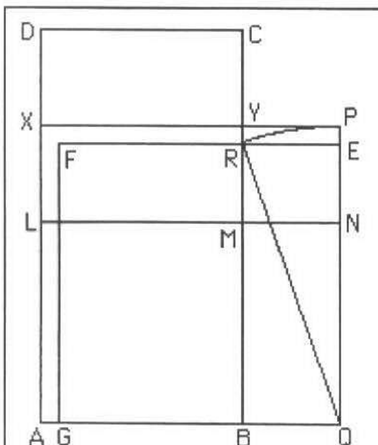


The Katyayana form of Pythagoras's theorem



To find a square equal to the sum of two given squares - given in all Sulbasutras

the version as it appears in the Baudhayana Sulbasutra.



Finding a square equal to a given rectangle

Sulba Sutra ("code of the ropes for measuring the altar")

The code used is as follows:

The Sanskrit consonants

ka, ta, pa, and ya all denote 1;
kha, tha, pha, and ra all represent 2;
ga, da, ba, and la all stand for 3;
Gha, dha, bha, and va all represent 4;
gna, na, ma, and sa all represent 5;
ca, ta, and sa all stand for 6;
cha, tha, and sa all denote 7;
ja, da, and ha all represent 8;
jha and dha stand for 9; and
ka means zero.

Vowels make no difference and it is left to the author to select a particular consonant or vowel at each step. This great latitude allows one to bring about additional meanings of his own choice. For example kapa, tapa, papa, and yapa all mean 11. By a particular choice of consonants and vowels one can compose a poetic hymn with double or triple meanings. Here is an actual sutra of spiritual content, as well as secular mathematical significance.

gopi bhagya madhuvrata
srngiso dadhi sandhiga
khala jivita khatava
gala hala rasandara

While this verse is a type of petition to Krishna, when learning it one can also learn the value of pi/10 (i.e. the ratio of the circumference of a circle to its diameter divided by 10) to 32 decimal places. It has a self-contained master-key for extending the evaluation to any number of decimal places.

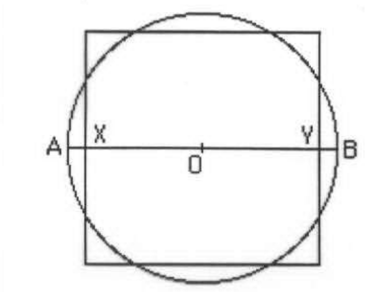
The translation is as follows:

> O Lord anointed with the yogurt of the milkmaids' worship (Krishna), O savior of the fallen, O master of Shiva, please protect me.

At the same time, by application of the consonant code given above, this verse directly yields the decimal equivalent of pi divided by 10: pi/10 = 0.31415926535897932384626433832792. Thus, while offering mantric praise to Godhead in devotion, by this method one can also add to memory significant secular truths.

Circling the square, and vice versa

All the Sulbasutras contain a method to square the circle. It is an approximate method based on constructing a square of side $^{13}/_{15}$ times the diameter of the given circle as in the diagram on the right. This corresponds to taking $\pi = 4 \times (^{13}/_{15})^2 = {}^{676}/_{225} = 3.00444$ so it is not a very good approximation and certainly not as good as was known earlier to the Babylonians.

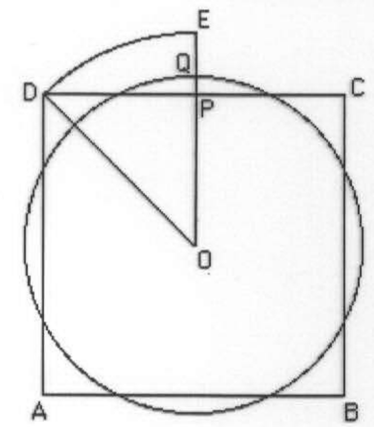


The Sulbasutras 13/15 method of "circling the square"

The Sulbasutras also examine the converse problem of finding a circle equal in area to a given square.

A cord of length half the diagonal of the square is stretched to the East. With one-third of the part lying outside added to the remainder of the half diagonal, the requisite circle is drawn.

This radius is 0.569 (if the side is 1), its area is 1.017.



The Sulbasutras method of "circling the square"

To transform a circle into a square, the diameter is divided into 8 parts. One such part, after being divided into 29 parts is reduced by 28 of them and further by the sixth of the part left less the eigth of the sixth part.

The side of the square is $1 - (1/8)\{1-(1/29)[1-(1/6)(1-1/8)]\}$

This is 0.87868... and the area is 0.7720816, instead of the correct value 0.7853

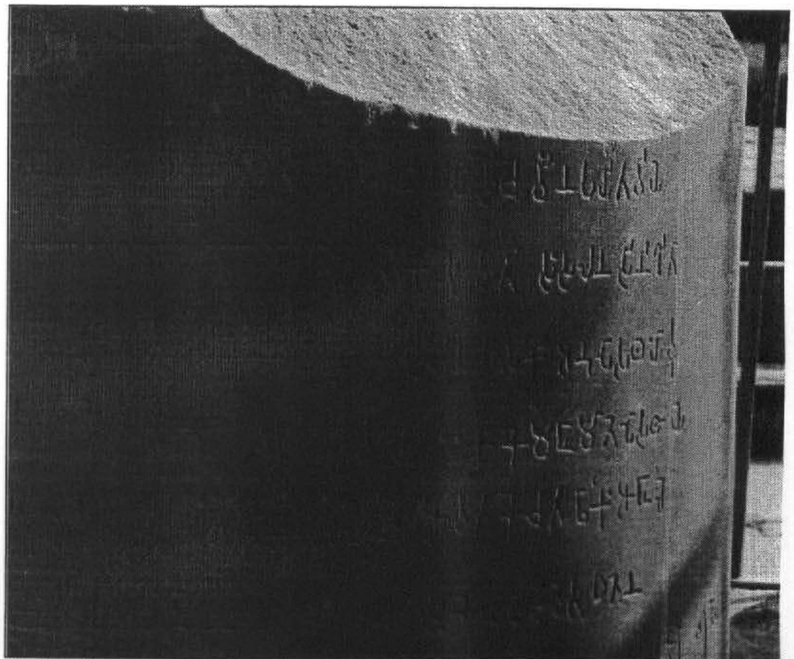
Values of π

It is worth noting that many different values of π appear in the Sulbasutras, even several different ones in the same text. This is not surprising for whenever an approximate construction is given some value of π is implied. The authors thought in terms of approximate constructions, not in terms of exact constructions with π but only having an approximate value for it.

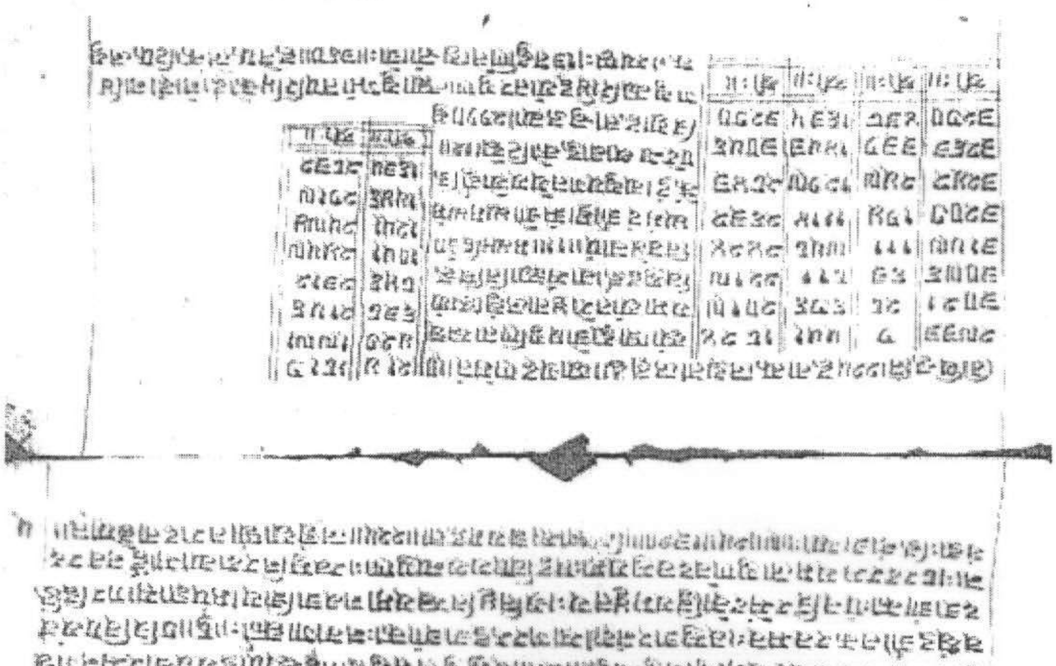
In the Baudhayana Sulbasutra, as well as the value of $^{676}/_{225}$, there appears $^{900}/_{289}$ and $^{1156}/_{361}$.

In different Sulbasutras the values 2.99, 3.00, 3.004, 3.029, 3.047, 3.088, 3.1141, 3.16049 and 3.2022 can all be found. The value $\pi = {}^{25}/_{8} = 3.125$ is found in the Manava Sulbasutras.

ARYABHATA 500 AD



Ashoka Pillar, Sarnath Archaeological Park, Sarnath. 243 BC



Brahmagupta's manuscript 630 AD

After 700 C.E. another notation, called by the name "Indian numerals," which is said to have evolved from the brahmi numerals, assumed common usage, spreading to Arabia and from there around the world. When Arabic numerals (the name they had then become known by) came into common use throughout the Arabian empire, which extended from India to Spain, Europeans called them "Arabic notations," because they received them from the Arabians. However, the Arabians themselves called them "Indian figures" (Al-Arqan-Al-Hindu) and mathematics itself was called "the Indian art" (hindisat).

Evolution of "Arabic numerals" from Brahmi (250 B.C.E.) to the 16th century.