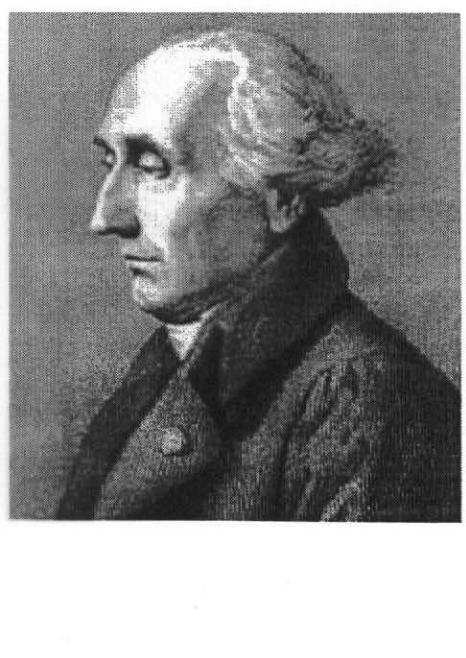
Jean Le Rond d'Alembert (1717-1783)

Son of an artillery officer and a nun, abandoned on the steps of the church of St Jean Le Rond.
d'Alembert was turned off to the study of theology in College
In 1735-1741, careers as lawyer and medical doctor

He believed mechanics to be based on metaphysical principles and not on experimental evidence.
In 1744, he gave an alternative approach to fluid flow
In 1747, wrote about vibrating strings. The first appearance of the wave equation in print but suffers from the defect that he used mathematically pleasing simplifications of certain boundary conditions which led to results which were at odds with observation.
In 1746, started writing an Encyclopedia with Diderot
In 1751, dispute with Euler & stopped publishing papers

In the 1754 article *Différentiel* in volume 4 of *Encyclopédie* , he suggested that the theory of limits be put on a firm f oundation. He was one of the first to understand the importance of functions and, in this article, he defined the derivative of a function as the limit of a quotient of increments of slopes of secant lines.
Definition (d'Alembert): One magnitude is said to be the limit of another when the second may approach the first within any given magnitude, however small.

Joseph Louis Lagrange (1736-1813)



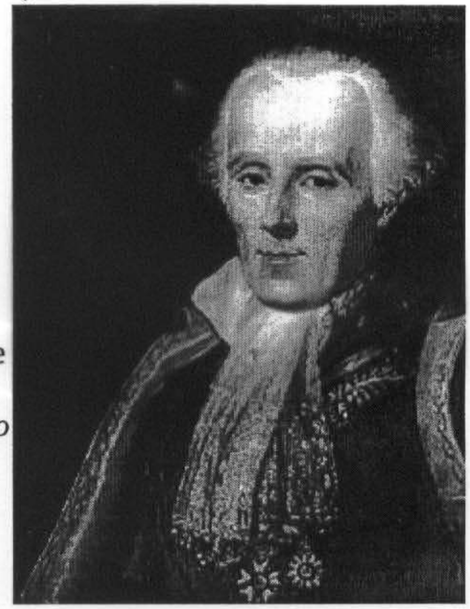
Born in Torino (now Italy) to a wealthy family
Starting in 1754, he developed the *Calculus of variations*
In 1766, recommended by Euler and D'Alembert, Lagrange succeeded Euler as the director of mathematics at the Prussian Academy of Sciences in Berlin.
In 1772, discovered *Lagrangian points* for 3-body problem
In 1773 proved *Wilson's Theorem*: (n-1)!+1 divisible by p
In 1786, Frederick the Great died, and Lagrange, gladly accepted the offer of Louis XVI to migrate to Paris.
In 1793, asked to head the committee on weights&measures which in 1799 accepted the decimal system.
Between 1772 and 1788, Lagrange re-formulated Newtonian mechanics to simplify formulas and ease calculations. These mechanics are called Lagrangian mechanics.

Major book (1778): *Mécanique analytique*

Pierre-Simon Laplace (1749-1827)

Came from a prosperous farming family in Normandy (French coast)
Age 19, quit school and went to Paris; given a position as mathematics professor at the École Militaire
1770-1773 wrote 13 papers, differential equations and probability theory, finally got a job in Paris with the Académie des Sciences
In 1784 Laplace passed a student named Napolean Bonaparte
In 1790, part of committee to establish metric system
In 1793-95, fled Paris during Reign of Terror
In 1795, published *A philosophical essay on probabilities*
In 1799 published the *Traité de Mécanique Céleste*
In 1799, after only six weeks, Napolean removed Laplace from the office of Minister of the Interior, which he held:-

> *... because he brought the spirit of the infinitely small into the government.*



Adrien-Marie Legendre (1752-1833)

In 1770, at the age of 18, Legendre defended his (honors) thesis in mathematics and physics
From 1775 to 1780 he taught with Laplace at École Militaire

In 1783, He gave a proof of a result due to Maclaurin, that the attractions at an external point lying on the principal axis of two confocal ellipsoids was proportional to their masses. He then introduced what we call today the Legendre functions and used these to determine, using power series, the attraction of an ellipsoid at any exterior point.
In 1785, wrote a paper on number theory containing "Gauss'" quadratic reciprocity – Legendre's proof was unsatisfactory.
In 1790, part of committee to establish metric system
In 1793-95, fled Paris during Reign of Terror

In 1806, developed the least squares method to fit data
In 1811/1817 published *Exercices du Calcul Intégral*
Elliptic integrals, Gamma functions
1800-1830 tried and failed to PROVE the parallel axiom

Blaise Pascal (1623-1662)

Moved to Paris age 8, wrote a short treatise on the sounds of vibrating bodies age 11, forbidden to study mathematics by his father. After a year, he was allowed to sit in on Mersenne's monastic seminar and listen to Descartes, Roberval, and other scientists of the day.
He died at age 39, possibly of cancer.

Age 16: proved Pascal's Theorem in Plane Geometry: it states that if a hexagon is inscribed in a circle (or conic) then the three intersection points of opposite sides lie on a line

Age 19 (1642) In an effort to ease his father's endless, exhausting calculations, and recalculations, of taxes owed and paid, Pascal, not yet nineteen, constructed a mechanical calculator capable of addition and subtraction.

Age 30 (1653): wrote *Traité du triangle arithmétique* ("Treatise on the Arithmetical Triangle"). This contained the "Pascal Triangle" (used by Newton in 1665 to get binomial theorem)

1654: Five letters to Fermat, establishing the theory of probability. (1) How many throws of dice do you expect before rolling 12? (2) A gambler friend, the Chevalier de Mere, wrote Pascal: how to split the pot if the game is interrupted. What is probability of W wins before L losses? Solution uses the Pascal triangle.

1657: Pascal's wager: "If God does not exist, one will lose nothing in believing in him, while if he does exist, one will lose everything by not believing."

Marin Mersenne (1588-1648)

Mersenne numbers are those of the form $M_n = 2^n - 1$

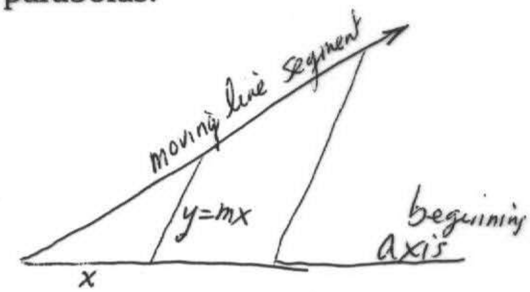
The name is because Mersenne tabulated them and pointed out which were prime for n<258. He omitted some (like n=61) and thought the 67[th] one was prime (corrected by Euler 1725)

Pierre de Fermat (1601-1655)

Went to University of Toulouse (SW France), then Bordeaux.
1630-31: studied civil law at the University of Orléans.
Given title of councillor at the High Court in Toulouse,

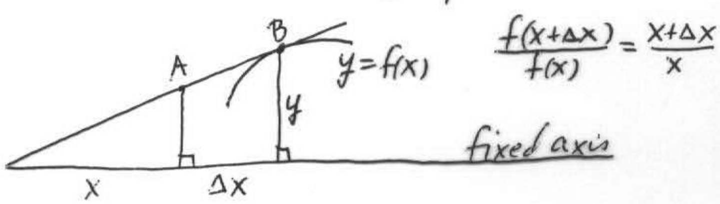
Ad Locos Planos et Solidos Isagoge (Introduction to plane and solid loci) was circulated in manuscript form in 1636, predating the publication of Descartes' famous *La géométrie*. This manuscript was published posthumously in 1679.

Fermat used only the x-axis and considered a perpendicular moving along the line, whose length y changed according to a formula. He showed that the equations $y=mx$ and $y=x^2+bx+c$ gave lines and parabolas.



1638

Fermat developed a method for determining maxima, minima, and tangents to various curves that was equivalent to differentiation. (Chapter 11)



In number theory

Most famous for Fermat's Last Theorem:t $x + y = z$ has no solutions if $n>2$.
He solved this for $n=3$ and $n=4$.
Fermat also studied Pell's equation ($y^2=dx^2+b$), Fermat primes (1+power of 2), ...
While studying perfect numbers he discovered Fermat's Little Theorem: $n!=-1 \pmod n$
He also invented the proof technique of infinite descent, an early form of induction.

Abraham de Moivre (1667-1754)

Studied in Colleges but never received a college degree.
1685, Henry IV's Edict of Nantes evicted Protestants from France,
Spent rest of his life in London
In 1710, part of Taylor's committee to referee Newton-Leibniz

De Moivre wrote a book, <u>The Doctrine of Chances</u>, (1710)
said to have been prized by gamblers.
First "published" in 1718

If event occurs a times and doesn't occur b times
$$p = \frac{a}{a+b} \quad \text{(odds are a to b)}$$

① How many trials for event to occur 50%?
$$p^n = 1/2 \qquad n = \frac{\log(2)}{\log(1/p)}$$

② Geometric Random Variable: $E(X)$, $X = \#$ of trials before event happens
$$E(X) = \sum_0^{\infty} n\, P(X=n) = \sum n p(1-p)^{n-1} = 1/p$$

③ <u>Law of Large Numbers</u>: after n trials
middle term $M = \binom{n}{n/2}$ has probability $\frac{M}{2^n} \longrightarrow \left(\frac{2e}{\sqrt{2\pi}}\right)\frac{(n-1)^n}{n^n\sqrt{n-1}} \approx \sqrt{\frac{2}{\pi n}}$

<u>Example</u>: $n=100$: middle (50) occurs 7.98 times

④ $P\left(X=\frac{n}{2}+t\right) \approx \sqrt{\frac{2}{\pi n}}\, e^{-(2t^2/n)}$ for $t \ll \frac{n}{2}$ (inflection at $t=\frac{1}{2}\sqrt{n}$)

$$P\left(\frac{n}{2} \le X \le \frac{n}{2}+k\right) \sim \sqrt{\frac{2}{\pi n}} \int_0^k e^{-(2t^2/n)}\,dt$$

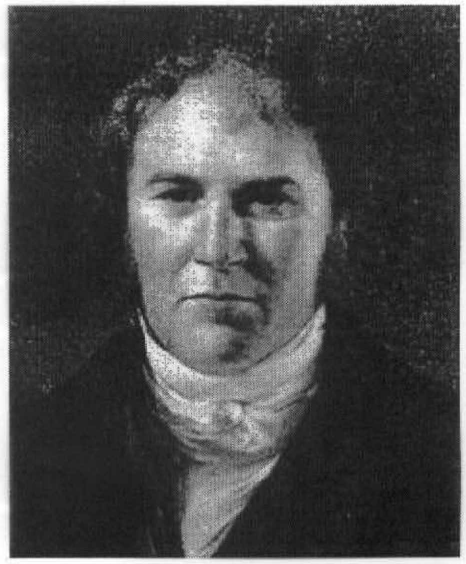
Bell Curve

(Jacob) <u>Bernoulli</u>: How many trials to determine p within <u>moral certainty</u> (99.9%)
<u>Bernoulli</u>: 25,550 trials to morally certain p is within 2%
<u>De Moivre</u>: 6,500 trials suffice

$$P\left(\left|\frac{X}{N}-p\right|<\varepsilon\right) > 1-\frac{1}{c}$$

Robert Adrain (1775-1843)

He was born in Carrickfergus, Ireland but emigrated to the USA in 1798 after a failed uprising.
Had a number of jobs at private (high school) academies (Princeton and York PA)



When the first mathematics journal, the *Mathematical Correspondent,* began publishing in 1804

At Rutgers:

Queen's College was closed 1795-1807 for lack of money
From 1809 to 1813, the only professor at Queen's College
(professor of "Mathematics and Natural Philosophy")
Adrain actually taught all of the upper level subjects at Queen's except Moral Philosophy and Composition, which Condict still taught. Freshmen and Sophomores were still taught by a tutor.

In 1809, while analyzing errors in surveying and dead reckoning at sea, Adrain discovered and gave the first rigorous proof of the Gauss Distribution for errors in Probability Theory, unaware that Laplace had claimed to be able to prove it in 1805.

The classes used notes from Adrain's American edition of Hutton's *A Course in Mathematics*. Consisting mostly of rules to memorize without explanation (like all British texts of the time), the first 400 pages covered computations of square roots (up to 9 decimal places), logarithms, solutions of cubic equations and trigonometry. The last 100 pages of this book, written by Adrain, consists of applications to surveying (trig), brickwork (volume), carpentry and masonry (surface area), plumbing, etc. Copies of Adrain's 1816 edition of Hutton's book are preserved in the Special Collections at Alexander Library.

During 1816-1825, Queen's College closed again for lack of money. It reopened in 1825 and was renamed ***Rutgers College***. Robert Adrain was rehired as the Professor of Mathematics, at an annual salary of $1,750. His decision to move back from New York City was influenced by his wife's health and the fresh country air in New Brunswick!
He left for Penn in 1827-1834, but returned to New Brunswick where he died in 1843

1) Symmetric about 0
2) $\phi(x)$ goes to 0 at $\pm\infty$
3) $\int \phi(x)\,dx = 1$
4) arithmetical mean of μ, σ^2 is most probable value

$$\phi(x) = \frac{h}{\sqrt{\pi}} e^{-h^2x^2}$$

Central Limit Theorem: any mean is normally distributed for $n >> 30$