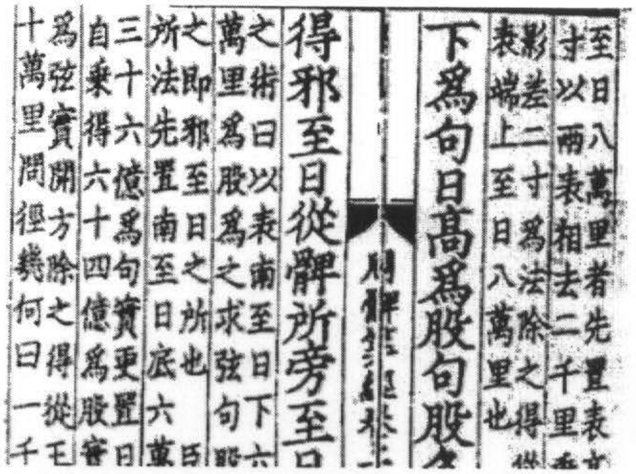
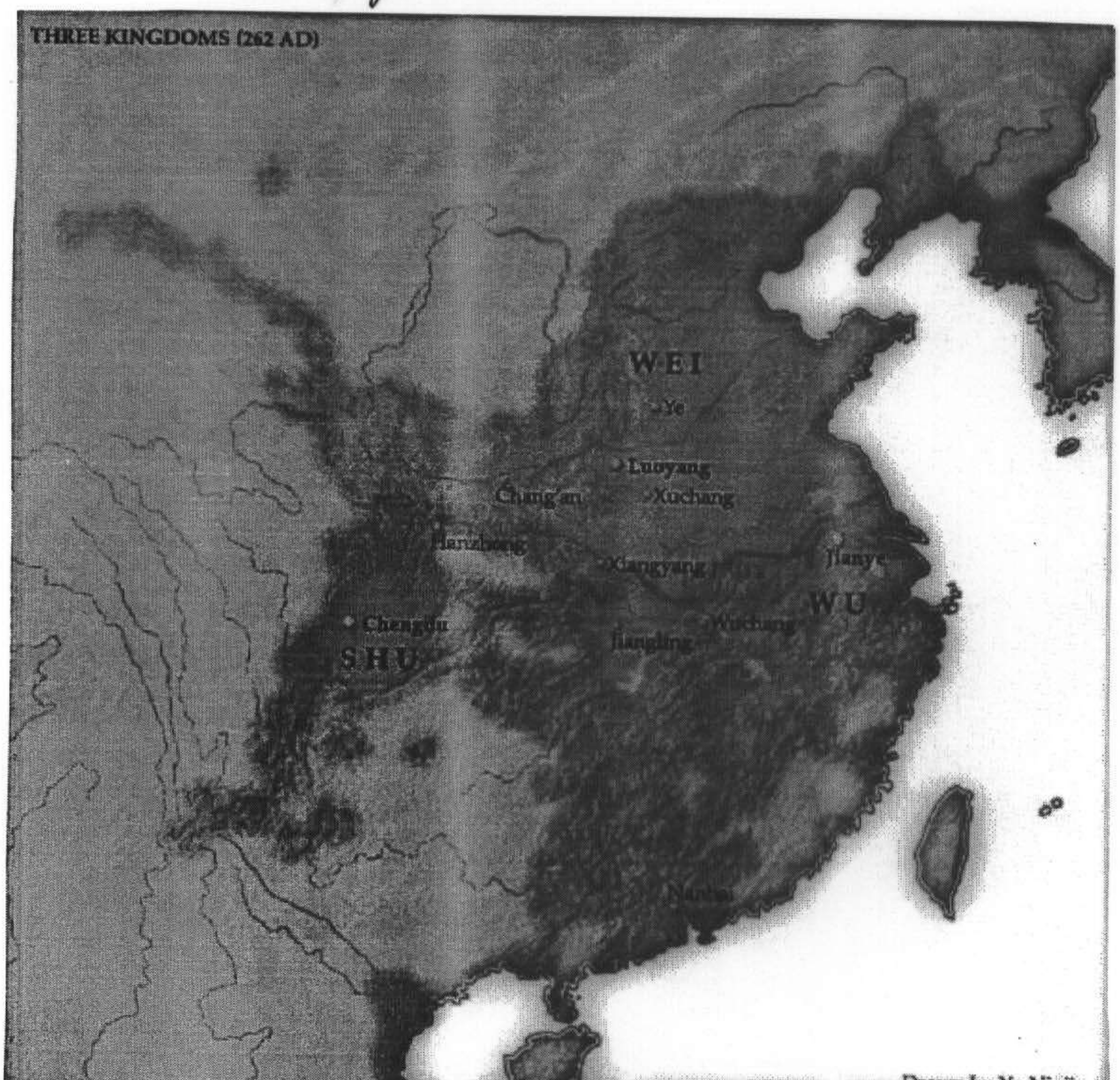


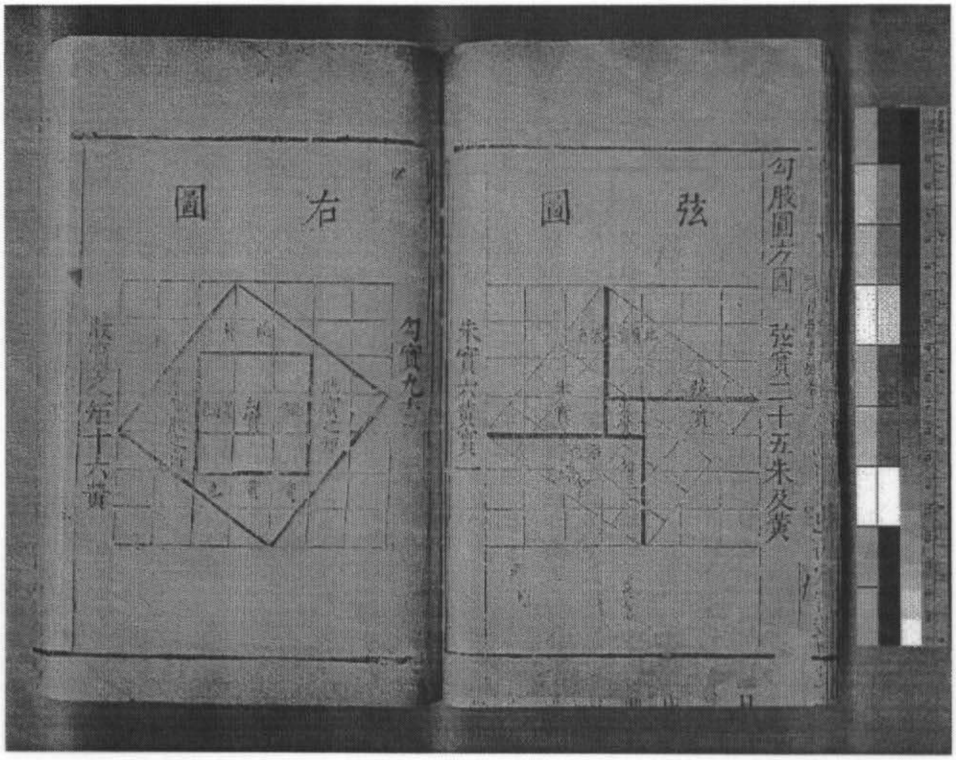
至日八萬里者先置表
寸以兩表相去二千里
影差二寸爲法除之得
表端上至日八萬里也
下爲句日高爲股句股
周髀算經卷上
得邪至日從髀所旁至
之術日以表南至日下六
萬里爲股爲之求弦句
之即邪至日之所也
所法先置南至日底六
三十六億爲句實更置
自乘得六十四億爲股
爲弦實開方除之得從
十萬里問徑幾何曰一

Zhao Shuang (220-280 AD)
Three Kingdoms Period (Wei, Shū, Wú)



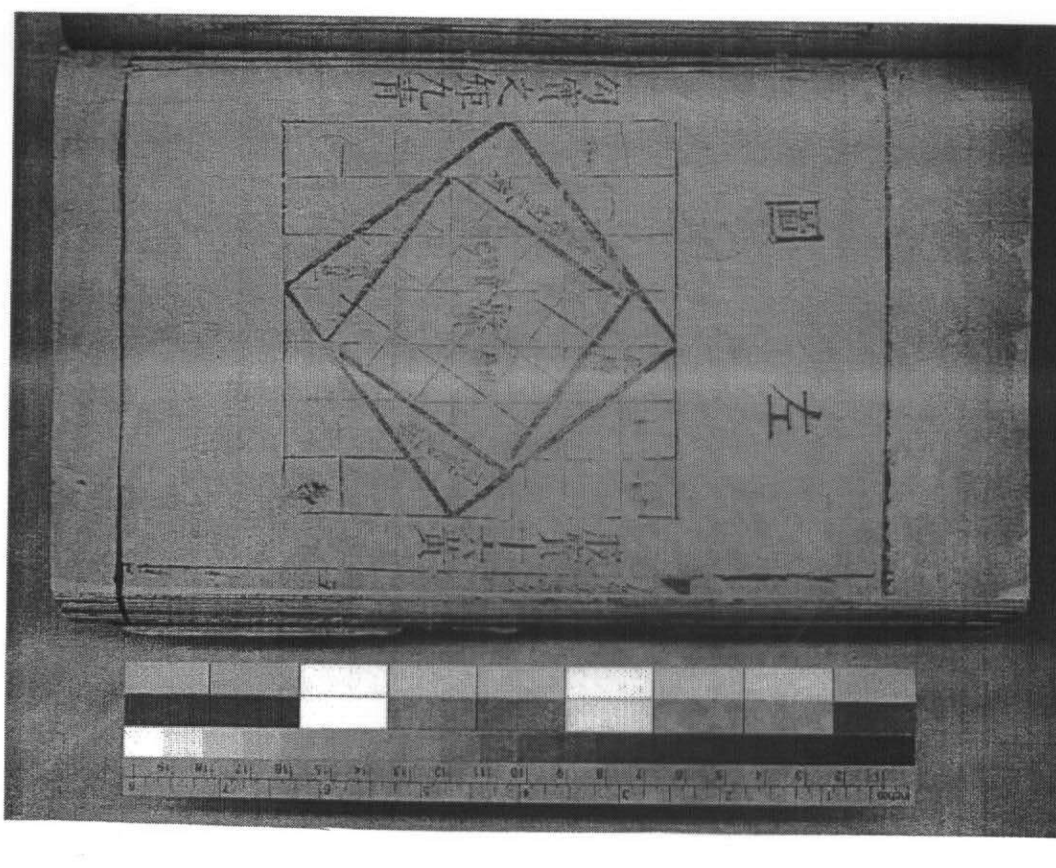
Zhoubi suanjing

These two pages are from the *Zhoubi suanjing* (*Arithmetical Classic of the Gnomon and the Circular Paths of Heaven*), a Chinese book on astronomy and mathematics dated to approximately 100 BCE. These images are from a Ming dynasty copy printed in 1603. These diagrams were added to the original text at some point an attempt to illustrate a dissection proof of the "Pythagorean Theorem", known by the Chinese as the *Gougu* theorem. A complete English translation and analysis of the *Zhoubi suanjing* is given by Christopher Cullen in his Astronomy and mathematics in Ancient China: the Zhou bi suan jing (Cambridge University Press, 1995). See, in particular, appendix one.



100 BC

On this page, the diagram on the right is usually called the "hypotenuse diagram" and illustrates the proof of the theorem in the 3-4-5 case. The diagram on the left shows how a square of side 3 fits into a square of side 5.



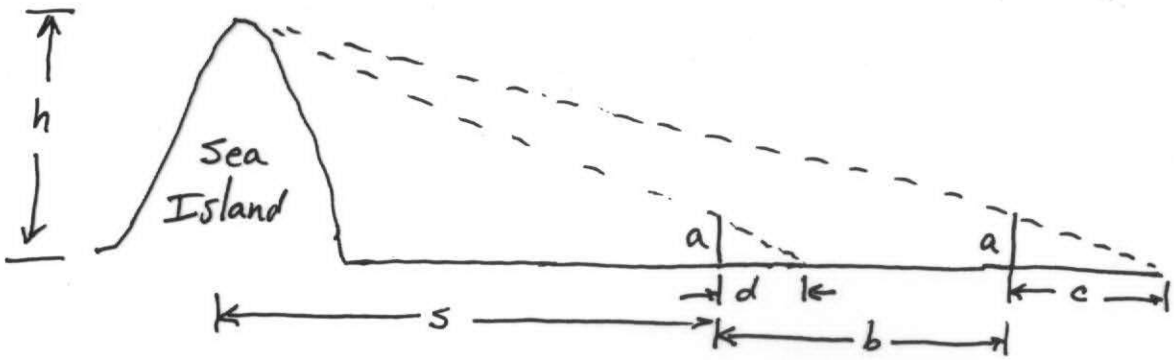
淳風等謹依密率爲田
百五步八十八分步之
曰半周半徑相乘得積步
從相乘爲積步也假令圓
六弧之一面與圓徑之半
率一而弧周率三也又按
一面乘六弧半徑二因而
之冪若又割之以以十二
弧之半徑四因而六之則
冪割之彌細所失彌少割
不可割則與圓周合體而
之外猶有餘徑以面乘徑
夫弧之細者與圓合體則
餘徑則冪不外出矣以一
裁之每輒自倍故以半周
冪此以周徑謂至然之數
率[illegible][illegible]三者從其六弧之

Liu Hui

Liù Hui (c.250 AD) (Wei Kingdom)

Sea Island Math Manual (Haidao suanjing)

Calculating Methods ("Shu")



$a = 5$ feet

$b = 1000$ feet

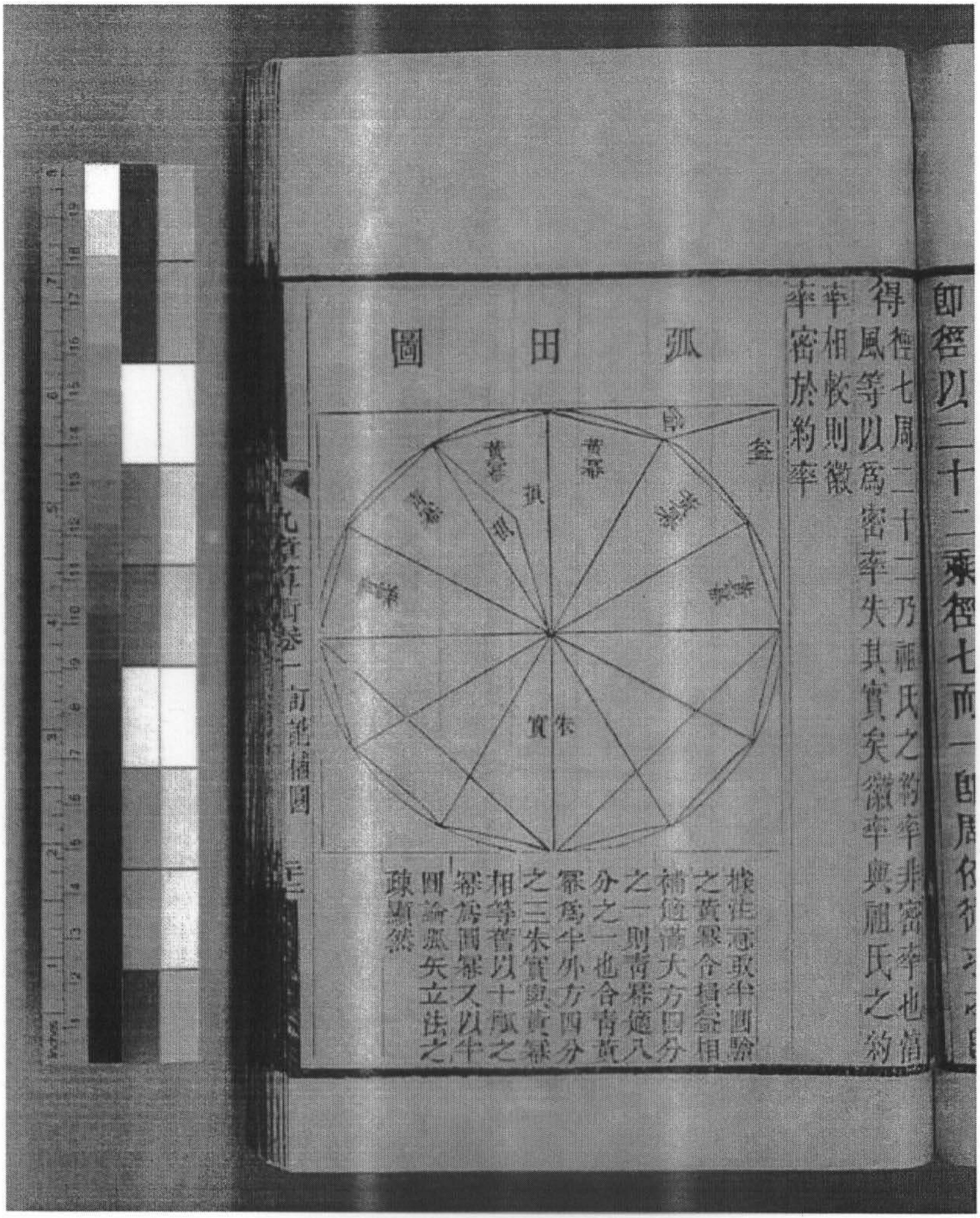
$c = 127$ feet

$d = 123$ feet

$$h = a + \frac{ab}{c-d}$$

$$s = \frac{bd}{c-d}$$

Method of double differences



Jiuzhang Suanshu
"Nine Chapters on the Mathematical Art"
Liu's method of computing π as 3.14024
Zu Chongzhi (≈ 450AD) extended to 3.1415927

Ming Dynasty copy
Original ≈ 0 AD?

"Chinese Remainder Theorem" (400 AD $\pm$)

Sunzi Suanjing

Nothing is known about the author except his text *Sunzi Suanjing*(Sun Zi's Mathematical Manual). It is generally believed that the book was completed in the fourth or the fifth century.

The*Sunzi Suanjing*consists of three chapters, the first describing systems of measuring with considerable detail on using counting rods to multiply, divide, and compute square roots. The second and third chapters consist of problems (28 and 36 respectively) concerning fractions, areas, volumes etc. similar to, but rather easier than, the problems in the*Nine Chapters on the Mathematical Art*. One problem, however, is of special interest, this being Problem 26 in Chapter 3:

Suppose we have an unknown number of objects. When counted in threes, 2 are left over, when counted in fives, 3 are left over, and when counted in sevens, 2 are left over. How many objects are there?

$X \equiv 2 \mod 3$

$X \equiv 3 \mod 5$

$X \equiv 2 \mod 7$

This, of course, is important for it is a problem which is solved using the Chinese remainder theorem. In fact the solution given, although in a special case, gives exactly the modern method. After solving the particular problem (the answer is 23) the Sunzi Suanjing gives a method for arbitrary remainders:

Multiply the number of units left over when counting in threes by 70, add to the product of the number of units left over when counting in fives by 21, and then add the product of the number of units left over when counting in sevens by 15. If the answer is 106 or more then subtract multiples of 105.

(X only well defined mod 105)

$$
\begin{array}{lll}
70 \equiv 1 \mod 3 & 21 \equiv 0 \mod 3 & 15 \equiv 0 \mod 3 \\
\equiv 0 \mod 5 & \equiv 1 \mod 5 & \equiv 0 \mod 5 \\
\equiv 0 \mod 7 & \equiv 0 \mod 7 & \equiv 1 \mod 7
\end{array}
$$

so $X = 2*70 + 3*21 + 2*15 = 233$

$$
\begin{array}{r}
233 \\
-210 \\
\hline
\equiv 23 \mod 105
\end{array}
$$

Polynomial Equations

① . Han Dynasty : Square Root Algorithm

$$\sqrt{55,225} = 100a + 10b + c$$

$$= 200 + 10b + \text{lower term}$$

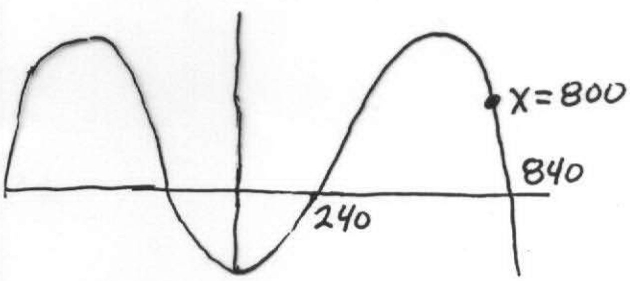
$$(200+10b)^2 = 40,000 + 4,000b + 100b^2 + \text{l.o.t.}$$

$$15,225 = 4000b + \text{hopefully smaller terms}$$

Try $b = 3$.

② Song Dynasty : Solve polynomials the same way
Details taken from 1247 document of Qin Jiushao

$$f(x) = -x^4 + 763,200x^2 - 40,642,560,000 = 0$$



Guess $x = 800$

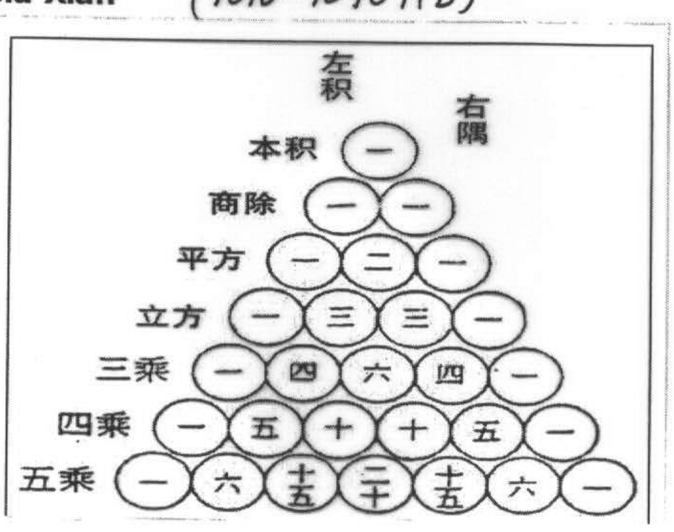
Evaluate $f(800+y)$

$$-y^4 - 3200y^3 - (3076800)y^2 - (\overset{C}{\cdots})y + (\overset{D}{\cdots}) = 0$$

Guess $y = 40$, evaluate $f(840+z)$, etc.

Synthetic division: $f(x) = (x-800)\{(x-800)[(x-800)\{(800-x)-3200\}-(307\cdots)]-C\}+D$

Jia Xian (1010–1070 AD)



Expansion of $(800+y)^4$ etc. using "Pascal Triangle" (France, 1650)