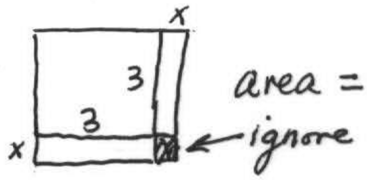
Find $\sqrt{10}$: Guess 3



area = 10

$$3^2 + 6x = 10$$
$$6x = 1$$
$$x = 1/6 \ (0;10)$$

$$(3\tfrac{1}{6})^2 = \frac{361}{36} = 10.028$$

False Position:

YBC 4652: I found a stone but did not weigh it; after I added one-seventh and then one-eleventh, it weighed 1 mina. What was the original weight of the stone?

x = weight in mina ($\cong$ 1 pound)

Answer: $x = 48\tfrac{1}{8}$ gin (60 gin equals one mina)
A 'gin' is also called a shekel.

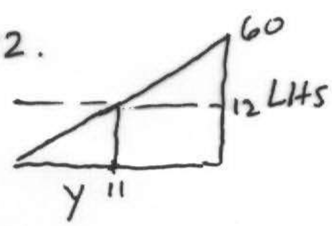
Modern Solution

$$\underbrace{\left(x + \frac{x}{7}\right)}_{y} + \frac{1}{11}\left(x + \frac{x}{7}\right) = 60 \text{ gin, or } y = x + \frac{x}{7},\ y + \frac{y}{11} = 60$$

1. Guess $y = 11$. Then LHS $= 12$.

   $y : 11 = 60 : 12 = 5$

   so $y = 55$



$$y = \frac{60 \cdot 11}{12} = 55$$

$$x = \frac{55 \cdot 7}{8} = 48\tfrac{1}{8}$$

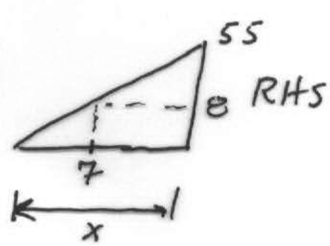
$1/8 = 0;7,30$

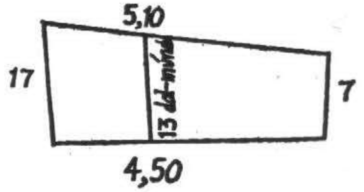
$7/8 = 0;52,30$

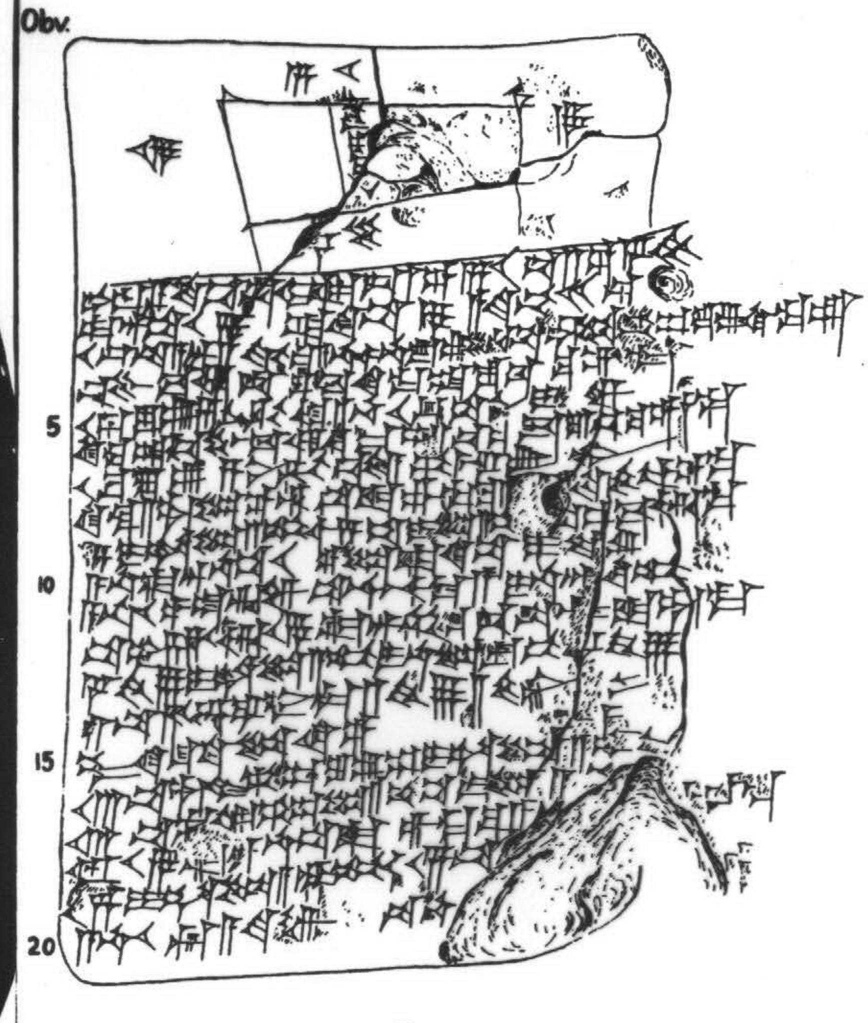
2. Guess $x = 7$. Then RHS $= 8$

   so $x : 7 = 55 : 8 = 6\tfrac{7}{8}$

   so $x = 7(6;52,30)$
   $= 42 + (6;4) + (0;3,30)$
   $= (48;7,30)$







B
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YBC 4675

Square Root Tables

No. 30. Plimpton 318. Single table of square roots.

From	15	30	to	54,9	57
	16,1	31		56,4	5[8]
	17,4	32		5[8],1	59
	etc.			1,2,3,2,1	1,2,1

The numbers in the last line are the squares of 1,1,1 and 1,1.[94]

No. 31. CBS 8270. Fragment containing five lines of a table of square roots. Reverse empty. Terminology destroyed.

Only the left-hand side of the fragment is preserved:

45,[4	corresponding to	52
46,4[9		53
48,3[6		54
50,25		55
52,[1]6		56

$0;15 = \frac{1}{4} \quad 0;30 = \frac{1}{2} = \frac{30}{60}$

$0;16,1 = \frac{961}{60^2} = \left(\frac{31}{60}\right)^2$

$0;17,4 = \frac{1024}{60^2} = \left(\frac{32}{60}\right)^2$

...

$0;54,9 = \frac{3249}{60^2} = \left(\frac{57}{60}\right)^2$

$0;45,4 = \frac{45}{60} + \frac{4}{60^2} = \frac{2704}{60^2} = \left(\frac{52}{60}\right)^2$

...

$0;50,25 = \frac{50}{60} + \frac{25}{60^2} = \frac{3025}{60^2} = \left(\frac{55}{60}\right)^2$

f. Logarithms

Tablets which contain tables of exponents a^n, where n is an integer between 2 and 10, and a is one of the numbers 9, 16, 1,40, 3,45 (note that all of these are squares), are known.[96a] We now have an Old-Babylonian tablet which answers the question: to what power must a certain number a be raised in order to yield a given number? This problem is identical with finding the *logarithm* to the base a of a given number.

[96] Cf. Neugebauer [4].

[96a] Cf. MKT I pp. 77ff., Neugebauer [6], and Neugebauer, Vorlesungen, pp. 199–202.

One side of the text in question (**MLC 2078**) is destroyed except for slight traces; all edges are preserved. On the other side and on the left margin appears the following:

1 1 15-e 2 íb-si$_8$
2 30-e 4 íb-si$_8$
3 45-e 8 íb-si$_8$
4 1-e 16 íb-si$_8$
5 *ga-mi-ru-um* níg (or: 4) *i du*(?) *uk* PI(?) . . . *ma*(?) . . . *i du*(?) *uk*(?)

2 6 2-e 1 íb-si$_8$
7 4-e 2 íb-si$_8$
8 8-e 3 íb-si$_8$
9 16-e 4 íb-si$_8$
10 32-e 5 íb-si$_8$
11 1,4-e 6 íb-si$_8$

Left Edge: 1,16[96b]-e 32 íb-si$_8$
1,30-e 1,4 íb-si$_8$ (64)

The meaning of the numbers in No. 1 is clearly

$$16^{0:15} = 2 \qquad 16^{0:30} = 4 \qquad 16^{0:45} = 8 \qquad 16^1 = 16,$$

$16^{1/4}, 16^{2/4}, 16^{3/4}$

or, in other words,

$$0;15 = \log_{16} 2 \qquad 0;30 = \log_{16} 4 \qquad 0;45 = \log_{16} 8 \qquad 1 = \log_{16} 16,$$

It is also evident that the two lines on the left edge are the direct continuation of this group, namely,

$$16^{1:15} = 32 \quad 16^{1:30} = 1,4 \quad \text{or} \quad 1;15 = \log_{16} 32 \quad 1;30 = \log_{16} 1,4.$$

Area

b. Trapezoids

The figure given on the obverse of **YBC 7290**[125i]

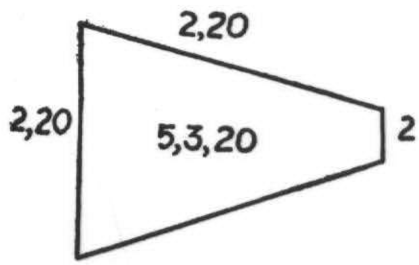


FIG. 4.

indicates that the area is obtained by

$$5;3,20 = 2;20 \cdot \frac{2;20 + 2}{2}.$$

On the reverse is given a trapezoid without inscribed numbers.

YBC 11126 is uninscribed on the reverse. The obverse gives the figure of a trapezoid with numbers.

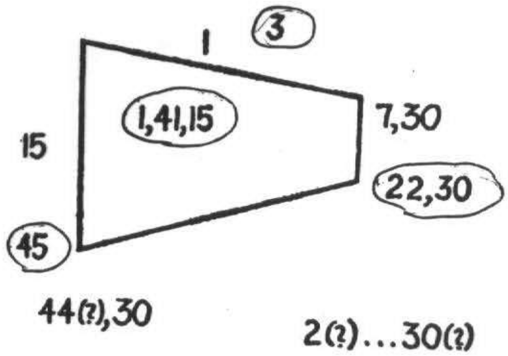


FIG. 5.

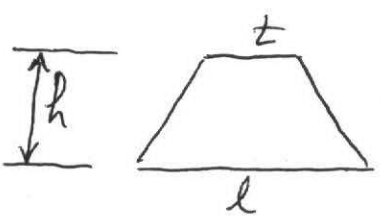
One set of these numbers satisfies the following relation

$$1,41,15 = 3,0 \cdot \frac{45 + 22;30}{2}$$

as expected for the area and sides. As for the remaining numbers, it is obvious that

$$45 = 3 \cdot 15 \qquad 22;30 = 3 \cdot 7;30 \qquad 3,0 = 3 \cdot 1,0$$

but the meaning of the coefficient 3 as well as the number 44(?),30 is not clear.



$$\text{area} = h\left(\frac{t+l}{2}\right)$$

c. Circle

YBC 7302[125j] gives the figure of a circle whose circumference $c = 3$. We therefore have $c^2 = 9$ and for the

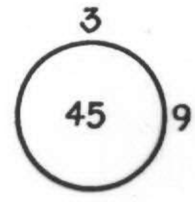


FIG. 6.

area

$$a = \frac{c^2}{4\pi} \approx \frac{c^2}{12} = 0;5 \cdot 9 = 0;45$$

using the value $\pi \approx 3$.[125k]

Analogously, we find in **YBC 11120**[125l]

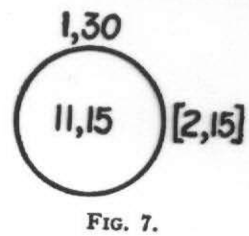
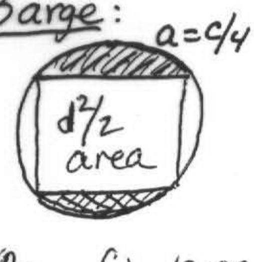


FIG. 7.

$$c = 1;30 \qquad c^2 = 2;15 \qquad a = 0;5 \cdot 2;15 = 0;11,15$$

$$C = \frac{3}{2} \quad c^2 = \frac{9}{4} \quad \text{area} \cong \frac{c^2}{12} = \frac{3}{16} = \frac{11}{60} + \frac{15}{60^2}$$

Barge: $a = c/4$



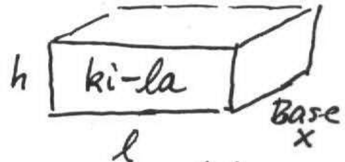
$d = c/3$

$$\begin{pmatrix}\text{Area of}\\ \text{circle}\end{pmatrix} - \begin{pmatrix}\text{area}\\ \text{of square}\end{pmatrix} = \frac{c^2}{12} - \frac{c^2}{18} = c^2/36 = 4/9\, a^2$$

"Area of barge is $\frac{2}{9}a^2$"

[125i] The tablet measures 7 by 8 cm.

[125j] The shape of the tablet is roughly circular; diameter 8 cm.; the reverse is uninscribed.



$E = 9$ gín

$t = 9$ days

wage $= 0;2$ gín per man-day

$m = 30$ workers

$\lambda = 0;10$ SAR/day per worker

$V = \dfrac{E \cdot \lambda}{\text{wage}}$

$= (270 \text{ man days})\left(\frac{1}{6} \frac{\text{SAR}}{\text{manday}}\right)$

$\boxed{V = 45 \text{ SAR}}$

$h = \frac{1}{2}$ GAR

$C = V/h = 90$ (or $1,30$)$/12$ square cubits

$C = \frac{90}{12} = 7\frac{1}{2}$ (or $7,30$)

c = area of base $= xl = x^2 + bx$

l = length $= x + b$ ($b = 3;30$ GAR)

$\boxed{x^2 + bx = c}$

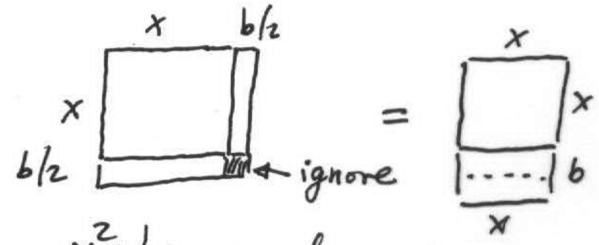
YBC 4657

8 [13]9 (gín) is the (total expenses in) silver of a ki-lá; the length exceeded the width by 3;30 (GAR);
[14]½ GAR is its depth, 10 gín (volume) the assignment, 6 še (silver) the wages.
[15]What are the length (and) the width? When you perform (the operations),
[16]take the reciprocal of the wages, multiply by 9, the (total expenses in) silver, (and) you will get 4,30;
[17]multiply 4,30 by the assignment, (and) you will get 45;
[18]take the reciprocal of ½ GAR (, the depth), multiply by 45, (and) you will get 7;30;
[19]halve that by which the length exceeded the width, (and) you will get 1;45;
[20]square 1;45, (and) you will get 3;3,45;
[21]to 3;3,45 add 7;30, (and) you will get 10;33,45;
[22]take its square root, (and) you will get 3;15;
[23]operate with 3;15 twice: add 1;45 to[170] the one,
[24]subtract 1;45 from[173] the other, (and) you will get the length and the width.
[25]5 GAR is the length; 1½ GAR is the width. Such is the procedure.

(1a)
$l = 5$ GAR length (uš)
$b = 1;30$ GAR width (sag)
$g = lb = 7;30$ SAR area (gagar) of the base
$h = 6$ kùš ($= 0;30$ GAR) depth (bùr)
$V = lbh = 45$ SAR volume (saḫar).

From the last relation it follows that the ki-lá is considered to be a prism. As for the work on the ki-lá, the following assumptions are made:

$\lambda = 0;10$ SAR work output to be expected daily for each worker (éš-kàr, translated "assignment")
$w = 6$ še $= 0;2$ gín wages per man per day (á) paid in silver
(1b)
$t = 9$ duration of the work in days (u₄)
$m = 30$ number of workers (erim-ḫá)
$M = 4,30$ number of man-days (erim-ḫá)
$E = 9$ gín total expenses in silver (kù-babbar).



$x^2 + bx = c$ = large area

$(x + b/2)^2 \cong c$ ignore $(b/2)^2$

$(x + b/2)^2 = c + (b/2)^2$

$\boxed{x = (-b/2) + \sqrt{c + (b/2)^2}}$