

ISRAEL MOISEEVITCH GELFAND

I spent many years in close association with Gelfand. Let me try to recall a few moments. My first “interaction” with him started under circumstances that were unfortunate for me. At seventeen I came to Moscow State University from the provinces to take the entrance examinations to the Department of Mechanics and Mathematics. I successfully passed my mathematics examinations, but failed an oral exam in physics. The same thing happened to a number of Jewish applicants, with the same few examiners. It was 1965, a rather tranquil time in the former Soviet Union. I don’t think that the examiners had any official instructions about whom to admit. They just followed their hearts. Nonetheless, the famous Moscow mathematician Alexander Semenovitch Kronrod let everyone know of this circumstance, and somebody told me that the news had reached Gelfand.

Calling a Corresponding Member of the Academy of Science without any prior introduction was a severe violation of the Soviet code of conduct. But my parents were so desperate that my father dared to call Gelfand, who of course did not know him. After hearing the story, Gelfand calmly replied: “Let the boy go the Pedagogical Institute (a sort of teachers’ college). If he is good enough he can also attend lectures at Moscow State.” I got the same recommendation from Kronrod, who also promised to supervise my studies in mathematics, and I followed this advice.

Kronrod (the last and perhaps most beloved student of N.N. Luzin) was a “problem solver”. He did not like theories and was not a fan of modern mathematics. Following an old Moscow tradition I spent my first semester on problems in real variables and in my second semester I was lured into seminars at Moscow State.

At one point Kronrod caustically asked me: “So what do you understand there?”

“Not much” - I confessed.

Kronrod shrugged his shoulders: “Then try the Gelfand Seminar. If you go to mathematical seminars just to pray you should do so in the main synagogue”. And so I started to attend the Seminar.

Gelfand’s Seminar has been already described in many places, both by participants and non-participants. Surprisingly, these descriptions often differ. There is not even agreement about the official starting time for the Seminar. I think that Simon Gindikin is right (he has given, in my opinion, the most accurate description of the Seminar): the official time was 6 to 8 p.m. But in fact, the seminar would start somewhere between seven or eight p.m., or even later, and go until eleven or so. Once or twice per semester Gelfand would start the seminar before seven, then tease the latecomers.

At the seminar I was immediately overwhelmed by Gelfand’s erudition and, at the same time, by the way he swooped down on the participants and speakers. Only a few, including Western foreigners and a grandson of A. N. Kosygin (the Soviet prime minister at that time), were shielded from Gelfand’s barbs. One of my advisers claimed that in fact Gelfand interrogated potential speakers in advance for hours and then demonstrated his pseudo-improvisations in public. I think the truth was a little bit of everything. Gelfand could also choose a speaker out of people currently present at the seminar without any warning.

From time to time Gelfand would formulate absolute truths at the seminar. For example, one of his most famous statements was: “Everything is Representation Theory”. Later I understood that such absolute truths were in fact relative: Gelfand’s truths depended heavily on the current situation, and their internal contradictions just created additional drama. Forty years later during a discussion of some noncommutative algebra problems he changed this famous statement to: “Nothing is Representation Theory”.

Once Gelfand started the seminar with a poll of distinguished participants Graev, Kirillov, Vishik and others (they all sat together at the front of the room): “Do you know derived categories? No? How can you do any mathematics without it?” After a few years the pendulum swung in the opposite direction. During a talk on orthogonal polynomials (at which only the senior people paid any attention to the speaker) Gelfand threw out the phrase: “Youngsters are the most conservative people in the world, they always know what is right and what is wrong.”

As I already noted, both speakers and participants were mercilessly ridiculed by Gelfand. However, victims could count on moral support from others. Gelfand’s jokes were not considered a stain on one’s reputation in any way. Rather, for a young person to be a target of Gelfand’s sarcasm was a sign of distinction.

It was clear that foreigners coming to the Seminar considered it a highly exotic adventure. I remember a talk by Lipman Bers. He mentioned a theorem by Maskit and added: “I am proud that Maskit is my former student”. Gelfand reacted immediately: “You cannot say ‘my former student’. This is like saying ‘my former son’.” This was not a linguistic, but rather a cultural difference: Gelfand always saw himself as a father figure to his students and collaborators.

Gelfand’s team always included both permanent and transient members. The most long-term collaborator of Gelfand (for almost sixty years) was M.I. Graev. Some of the team members would get well-posed problems, while others participated in discussions of rather vague ideas. In my own case it was often enough for Gelfand to ask: can we do something in this direction?

While working with the team, the only permanent Gelfand’s motto was: “I ask only simple questions”. His other reactions were sometimes unpredictable. From “Why are trying to adjust our project to your own interests? You do not see the big picture” to “I gave foolish advice and you just blindly followed it. It is enough to have just one fool in our company and it’s me in this case.”

After 1970, with the beginning of emigration, Gelfand’s team started changing more radically. Kazhdan, I believe, was the first to emigrate. Others followed his example. In my opinion, the greatest loss for Gelfand himself, as for all of us, was the departure of Joseph Bernstein.

Emigrants at that time disappeared completely behind the iron curtain, and we had a feeling that they were lost forever. It was hard. We, basically, did not have any social life outside mathematics. Our collaborators were usually our best friends, with whom we discussed everything: mathematics, politics, books, our personal lives, etc. Quiet and soft spoken Bernstein was open to everyone, especially young mathematicians. He would listen

to their vague and sometimes contradictory ideas and then often put them into brilliant and short statements.

The leader of the last Moscow team was Andrei Zelevinsky. He suggested to Gelfand that I be invited to work on their projects. Gelfand started our “negotiations” with a frontal attack. “Well, you are doing some homological algebra but we already have Beilinson for that. If you are going to work with me you have to start from scratch. In medieval times painter’s pupils worked for years just preparing paints for the master. Do you know what a hypergeometric function is? No? Very well, you can work with me on hypergeometric functions”.

After a few days Gelfand changed tactics. He asked me to open the celebrated hand-book of Bateman and Erdelyi and point out the formulas I like. He reacted to my choices quite positively: “Well, you have some taste. Why were you so interested in that abstract nonsense?”

Mostly, I worked with Gelfand in his apartment. The routine was almost always the same: I would do long calculations in my notebook while Gelfand talked on the phone with an endless stream of people. From time to time he would look over my shoulder to discuss my results.

One day the routine was broken. A son of Anatole Kouchnirenko was hit by a truck while riding a bicycle. In such situations access to the right hospital and the right treatment was the question of life and death. Despite official Soviet propaganda such things depended heavily on people’s connections, influence, bribes, etc. Gelfand called up one physician after another: “Can you help? What advice would you give? Which is a good hospital? Well, Doctor N. has a different opinion. Why? Is N. really good? How do you rate him in comparison with Dr. M.? And how would you rate yourself? OK, can we start a joint project together?”

At the end of the day the situation was somehow defused and Gelfand returned to mathematics. However, the medical discussion left some residue. “Do you understand the difference between Arnold, Manin and me”, Gelfand asked suddenly. “They are great coaches: they take talented students and train them to be Olympic champions. I can do all this, too, but I can also simply be a physiotherapist”.

In the Summer of 1988 or 1989 Gelfand lived with his daughter in a boarding house of the Academy of Sciences in Zvenigorod and we rented a summer house in Kratovo, not far from Zelevinsky, Feigin and Fuchs. The trip to see Gelfand was rather long: a train, a subway, then another train, then a bus. At the end of the trip I was pretty exhausted. So Gelfand suggested that we start something simple, such as looking for a correct theory of noncommutative determinants. The only hint I got from him that the theory must be based on Cramer’s rules. Gelfand had been asking about noncommutative determinants every semester since my sophomore year and my friends would always shrug their shoulders: “The old man is losing his grip. Who cares about such things now?”

At the boarding house, Gelfand lived in a tiny room. There was space only for two beds with their nightstands. We would kneel, with our notebooks on the beds, writing our formulas, and Gelfand would laugh like a happy child: “You just look at these formulas, they tell us what to do by themselves. How nice!”

We continued the game with formulas in Gelfand's kitchen in Moscow. Once a government official phoned Gelfand. He complained that his thirteen year old son hated mathematics and asked Gelfand for advice. I expected to hear a cascade of Gelfand's jokes, but he was dead serious. He asked the boy to pick up the phone and said: "I will give you just three problems: multiply one by one, one by negative one, and negative one by negative one".

The teenager gave the correct answer to the first two questions and then stopped. "That's great", Gelfand said, "you already know two thirds of all mathematics, you just need to try a little bit to get the rest of it". It was the best lesson in pedagogy I ever had.

My extended family emigrated to US in 1993. I spent a year as a visiting scholar at Harvard and Rutgers and then got my first teaching job at Oklahoma State. I was almost scared to death at that time: my knowledge of the American Midwest was limited to O. Henry's stories, and among Russians there circulated a lot of terrible tales about American students.

As usual, Gelfand defused the situation: "Are you a professional or not? As a professional you must be able to teach at elementary school, to give talks interesting to Harvard faculty, and everything in between. Just one piece of advice: make your students comfortable in your classes, one cannot teach without this."

After spending a year at Oklahoma State and another at Penn State, I returned to Boston. My wife and I rented a small apartment near Harvard Square. A memorial conference for Garrett Birkhoff was scheduled on the first of April that year and Gelfand was invited as a keynote speaker.

Gelfand came to Boston a few days early and we spent this time preparing his talk, on Lattice Theory. Unfortunately, a huge snowfall was expected in Boston right before the conference and the organizers at Harvard suggested cancelling the event. Many interested people would not be able to be there, the audience might be sparse, and so on. Gelfand surprised them by insisting that the show must go on under any circumstances. After all, no one worthy of being in the audience would miss it because of a little snow.

Gelfand's interests were at once intense and wide-ranging. Perhaps the most remarkable illustration of this is what happened on that snowy day he spoke at Harvard. To make the morning commute easier, Gelfand had spent the night in our apartment on a small futon in the living room. During our breakfast on April 1 my wife offered him some eggplant. "It is good", Gelfand said, "but the way you cook eggplant is totally wrong. I will teach you how to do it".

At that point I knew what might happen, and had to act to prevent a cooking lesson that might last several hours. I pleaded: "It's late". And in fact we started on our way through the snowdrifts.

Gelfand's talk, scheduled for one hour, lasted about two hours. Actually, he wanted to continue but the listeners began to be restless. After the talk Gelfand approached me: "Do you understand why I agreed to come? I did not know Birkhoff. It is all about lattices. Maybe they can replace category theory, which is too rigid".

We never returned to this subject and in a few month I left Boston for Arkansas. Gelfand did not call me very often, just three times a day. The first call was usually at 8 a.m.

After two years I started my tenure at the same university as Gelfand and the number of his everyday phone calls doubled.

Many thanks to Mark Saul and to my Rutgers colleagues whose remarks greatly improved the initial version of this story.