The Problem of Boltzmann Brains and How Bohmian Mechanics Helps Solving It

Roderich Tumulka

Department of Mathematics

RUTGERS UNIVERSITY

Tübingen, 13 August 2015

Joint work with Shelly Goldstein and Ward Struyve, supported by the John Templeton Foundation
arXiv:1507.08542 and 1508.01017
A “Boltzmann brain” is this: Let $M$ be the present macro-state of your brain. For a classical gas in thermal equilibrium, it has probability 1 that after sufficient waiting time, some atoms will “by coincidence” (or “by fluctuation”) come together in such a way as to form a subsystem in a micro-state belonging to $M$. That is, this brain comes into existence not by childhood and evolution of life forms, but by coincidence; this brain has memories (duplicates of your present memories), but they are false memories: the events described in the memories never happened to this brain!

Boltzmann brains are, of course, very unlikely. But they will happen if the waiting time is long enough, and they will happen more frequently if the system is larger (bigger volume, higher number of particles).
The problem is this: If the universe continues to exist forever, and if it reaches universal thermal equilibrium at some point, then the overwhelming majority of brains in the universe will be Boltzmann brains. According to the “Copernican principle,” we should see what a typical observer sees. Thus, the theory predicts that we are Boltzmann brains. But we are not. (Because most Boltzmann brains find themselves surrounded by thermal equilibrium, not by other intelligent beings on a planet.)

How can any of our serious theories avoid making this incorrect prediction?
It is expected (e.g., it is implied by ΛCDM) that the late universe will be close to de Sitter space-time, and the state of matter will close (in terms of local observables) to the “Bunch-Davies vacuum,” a quantum state invariant under the isometries of de Sitter space-time. The probability distribution it defines on configuration space gives > 99% weight to thermal equilibrium configurations, but positive probability to brain configurations, in fact > 99% probability to configurations containing brains if 3-space is large enough (in particular if infinite).

Question: Does this mean there are Boltzmann brains in the Bunch-Davies vacuum? What is the significance of this particular wave function for reality? Does a stationary state mean that nothing happens?

Or does the factual situation visit different configurations over time according to $|\psi|^2$?
Bohmian mechanics

For non-relativistic QM: [Slater 1923, de Broglie 1926, Bohm 1952, Bell 1966]

- Takes “particles” literally: electrons are material points moving in 3-space and have a definite position $Q_k(t)$ at every time $t$. There is also another physical object, mathematically represented by a wave function $\psi_t$ on configuration space $\mathbb{R}^{3N}$ ($N =$ no. of particles).

- Dynamical laws:

\[
\frac{dQ_k}{dt} = \frac{\hbar}{m_k} \nabla_k \text{Im} \log \psi(Q_1(t), \ldots, Q_N(t))
\]

\[
i\hbar \frac{\partial \psi}{\partial t} = H\psi
\]

- The law of motion (1) is equivalent to $dQ/dt = j/\rho$, where $Q = (Q_1, \ldots, Q_N)$ is the configuration, $\rho = |\psi|^2$ is the standard probability density, and $j$ is the standard probability current vector field in configuration space.

- Quantum equilibrium assumption:

\[
Q(t = 0) \text{ is random with distribution density } |\psi(t = 0)|^2.
\]
Example of Bohmian trajectories $Q(t)$: 2-slit experiment

wave-particle duality (in the literal sense)
Equivariance theorem: It follows that at any time, $Q(t)$ has distribution $|\psi(t)|^2$.

John S. Bell (1986):

De Broglie showed in detail how the motion of a particle, passing through just one of two holes in the screen, could be influenced by waves propagating through both holes. And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. [Speakable and unspeakable in quantum mechanics, page 191]
Freezing in Bohmian mechanics

- In Bohmian mechanics, there is the phenomenon of “freezing.”

**Theorem**

If $\psi$ is a non-degenerate eigenstate of $H$ then the Bohmian configuration does not move. (The same is true for Bohmian mechanics with particle creation.)

- That is because $H$ is real, so the conjugate of $\psi$ must be another eigenstate with the same eigenvalue, so $\psi$ must be real up to a global phase. As a consequence, Bohmian velocities (and particle creation rates) vanish.

- (Surprising because the momentun distribution is not concentrated on the origin. In Bohmian mechanics, momentum corresponds not to the instantaneous velocity but to the asymptotic velocity that the particle would reach if the potential were turned off.)

- It follows that, if non-relativistic Bohmian mechanics were true, and if the late universe were in a non-degenerate eigenstate, then the configuration would be frozen. Arguably, the Boltzmann brain problem is absent then.
There are two views on what situation exactly gives rise to a Boltzmann brain problem; they agree that a frozen universe does not have one.

The “optimistic” view insists that a Boltzmann brain will be problematical only if it is a functioning brain, at least for a short time. In a frozen universe, even if a Boltzmann brain configuration existed, it would not be functioning.

The “pessimistic” view retains the worry that a mere brain configuration may be problematical as it may encode all memories of the brain and perhaps the present thoughts. Then it becomes important that in a frozen universe, Boltzmann brain configurations cannot occur over and over (unlike in a classical gas in a box which, due to the eternal irregular motion, repeatedly assumes every configuration over time). If a Boltzmann brain configuration occurs once, then it stays forever and has the majority of observer-time since the normal brains are finite in number and lifetime. However, if 3-space is not extremely large, then the $|\psi|^2$ probability that a Boltzmann brain configuration occurs anywhere in space is tiny—so, with overwhelming probability, the problem is absent.
De Sitter space-time has metric

\[ ds^2 = dt^2 - e^{2Ht} \delta_{ij} dx^i dx^j \]

\(H = \) expansion speed = “Hubble parameter”

Simple quantum field theory: Hermitian scalar quantum field \(\varphi(x, t)\)

Wave functional \(\Psi(\varphi, t)\) on space of field configurations

common rescaling: \(dt = e^{Ht} d\eta\) (\(\eta = \) “conformal time”), \(y = e^{Ht} \varphi\)

\(-\infty < t < \infty\) but \(-\infty < \eta < 0\): \(t \to \infty\) corresponds to \(\eta \to 0\)

Schrödinger equation:

\[
i \frac{\partial \Psi}{\partial \eta} = \frac{1}{2} \int d^3x \left[ -\frac{\delta^2}{\delta y(x)^2} + \delta_{ij} \frac{\partial}{\partial y(x)} \frac{\partial}{\partial y(x)} \right] \Psi \\
+ \frac{i}{\eta} \left( \frac{\delta}{\delta y(x)} y(x) + y(x) \frac{\delta}{\delta y(x)} \right) \Psi
\]
Concrete Model

[Hiley and Aziz Mufti 1995; Pinto-Neto, Santos, and Struyve 2012]

- Bohmian model with field ontology: actual field configuration $\varphi(x, t)$

$$\frac{dy(x)}{d\eta} = \frac{\delta \text{Im} \log \Psi}{\delta y(x)} - \frac{1}{\eta}y(x)$$

- In terms of Fourier modes $y_k$, ($\mathbb{R}^{3+} = \text{half space, note } y_{-k} = y_k^*$)

$$i \frac{\partial \Psi}{\partial \eta} = \int_{\mathbb{R}^{3+}} d^3 k \left[ -\frac{\delta^2}{\delta y_k^* \delta y_k} + k^2 y_k^* y_k + i \frac{1}{\eta} \left( \frac{\delta}{\delta y_k^*} y_k^* + y_k \frac{\delta}{\delta y_k} \right) \right] \Psi$$

$$\frac{dy_k}{d\eta} = \frac{\delta \text{Im} \log \Psi}{\delta y_k^*} - \frac{1}{\eta}y_k$$
Freezing in the Bunch-Davies state

[Goldstein, Struyve, and Tumulka 2015]

- Bunch-Davies state: \( f = f_k(\eta) = \sqrt{1 + 1/k^2\eta^2}/\sqrt{2k} \)

\[
\psi = \prod_{k \in \mathbb{R}^3^+} \frac{1}{\sqrt{2\pi f}} \exp \left\{ -\frac{1}{2f^2} |y_k|^2 + i \left[ \left( \frac{f'}{f} + \frac{1}{\eta} \right) |y_k|^2 - \text{phase}(k, \eta) \right] \right\}
\]

- Solution to Bohmian eq. of motion: \( y_k(\eta) = \tilde{c}_k f_k(\eta) \) or

\[
\varphi_k(t) = c_k \sqrt{1 + k^2} \exp(-2Ht)/H^2 \quad (4)
\]

- Note that \( \exists \lim_{t \to \infty} \varphi_k(t) \) (freezing).

- In fact, at any time only the modes with wave lengths large compared to the Hubble distance \( 1/H \) are frozen. But the simple behavior (4) is as good as freezing for removing the Boltzmann brain problem: Too simple to support the complex behavior of a functioning brain (satisfying the “optimist”), and not giving rise to encoded memories that were not there initially (satisfying the “pessimist”).
Freezing in a generic state

- $\Psi$ will not be close to the Bunch-Davies state in Hilbert space. It will look locally similar, but Bohmian mechanics depends nonlocally on the wave function.

- Morally, for more or less any wave function and any initial field configuration, the asymptotic long-time behavior of $\varphi$ is

$$\varphi_k(t) \approx c_k \sqrt{1 + k^2 \exp(-2Ht)/H^2} \quad \text{for } t > t_0,$$

where $t_0$ is independent of $k$ (but depends on the wave function). In particular, $\exists \lim_{t \to \infty} \varphi_k(t)$.

[Ryssens 2012; Tumulka 2015]
Idea of derivation

- consider a single mode
- rescale field variable $z = \gamma(\eta)^{-1} y$
- rescale and phase-transform wave function,
  \[
  \Phi(z, \eta) = e^{\alpha(\eta) + i\beta(\eta)z^*z} \psi(\gamma(\eta) z, \eta)
  \]
- rescale time $d\tau = \gamma^{-2} d\eta$
- If scaling functions $\alpha, \beta, \gamma$ are chosen suitably, the evolution of $\Phi$ reduces to a non-relativistic Schrödinger equation in a harmonic oscillator potential
  \[
  i \frac{\partial \Phi}{\partial \tau} = - \frac{\partial^2 \Phi}{\partial z^* \partial z} + \omega^2 z^* z \Phi
  \]
  and non-relativistic Bohmian equation of motion
  \[
  \frac{dz}{d\tau} = \frac{\partial \text{Im} \log \Phi(z, \tau)}{\partial z^*}
  \]
  which do not become singular as $\tau \to 0^-$ or $\eta \to 0^-$ or $t \to \infty$.
- Thus, $\lim_{\tau \to 0^-} z(\tau)$ exists.
Thank you for your attention