1 Information on the course

1.1 About the instructor
My name is H. J. Sussmann. My office is Hill 538. My Rutgers phone number is 732-445-2390, extension 5407. My e-mail address is sussmann@math.rutgers.edu.

1.2 Web page
I have set up a Web page for our Math 300 section:

http://www.math.rutgers.edu/~sussmann/math300page-Fall12.html

All the instructor’s notes will be available there.

1.3 The lectures
We will have 26 lectures, on

• September 4, 6, 11, 13, 18, 20, 25, 27
• October 2, 4, 9, 11, 16, 18, 23, 25, 30
• November 1, 6, 8, 13, 15, 20, 27, 29
• December 4, 6, 11,

1.4 The midterms
There will be two midterm exams, on Tuesday, October 16 and Thursday, December 4.

1.5 Homework
Homework problems will be assigned each week on Tuesday or Thursday or both, as part of the notes posted in this Web page. All the homework assigned during one week will be collected on Thursday of the following week.
1.6 Your final grade

- Homework will count for about 24% of your grade.
- The two midterms will count—together—for about 36% (18% each).
- The final exam will count for the remaining 40%.

My office is Hill 538. My office hours will be:

- Monday, 1:00pm to 3:00pm, in my office,
- any other time (possibly including weekends), by appointment, in my office.

1.7 Textbook and notes

We will be using:

- the book *A Transition to Advanced Mathematics* (seventh edition), by Douglas Smith, Maurice Eggen, and Richard St. Andre;
- the notes written by the instructor.

The material of the instructor’s notes is an integral part of the course, as much as that of the book. Furthermore, the notes contain all kinds of important information. For example, in this set of notes there are lots of things you need to know in order to do your homework.

1.8 Always bring the book and the notes to class!

In the lectures, we are going to spend a lot of time looking at the book and the notes and analyzing definitions, arguments and proofs given there. So

Please always bring the book and the notes to class! You are going to need them.

1.9 Readings for the first 3 weeks (September 4, 6, 11, 13, 18, 20)

- the book’s “Preface to the student,”
- the book’s Chapter 1 (all of it!),
- the book’s Chapter 2, section 2.1,
- the instructor’s notes handed out during the first three weeks.
1.10  **Homework assignment no. 1, due on Thursday Sept. 13**

Before you start writing your homework, read carefully the rest of this handout, in particular §1.11 on “some remarks about mathematical writing.”

1. Book, Exercises 1.1 (pages 7-8-9): Problems 1 (non-starred items), 2 (non-starred items), 9 (non-starred items), 11 (non-starred items).

2. Book, Exercises 1.2 (pages 15-16-17-18): Problems 1 (non-starred items), 6 (non-starred items), and 16 (non-starred items).

3. (i) Prove or disprove: there exist integers \(x, y\) such that \(x^2 - y^2 = 28\).
   (ii) Prove or disprove: there exist integers \(x, y\) such that \(x^2 - y^2 = 29\).
   (iii) Prove or disprove: there exist integers \(x, y\) such that \(x^2 - y^2 = 30\).

   NOTE: to prove that there exists an object \(x\) such that a statement \(S(x)\) involving \(x\) is true, all you have to do is exhibit one. For example: to prove that there exists an integer \(x\) such that \(x^2 + 1 = 10\), you can just say:
   
   Let \(x = 3\).
   Then \(x\) is an integer.
   And \(x^2 + 1 = 10\).
   So \([[\text{there exists an integer } x \text{ such that } x^2 + 1 = 10]$$]

   To disprove the assertion that there exists an object \(x\) such that a statement \(S(x)\) involving \(x\) is true (i.e., to prove that there does not exist \(x\) such that \(S(x)\) is true), you can do it by showing that for every possible \(x\) the sentence \(S(x)\) is not true. For example, to prove that there does not exist a real number \(x\) such that \(x^2 + 1 = 0\), you can just say:
   
   Let \(x\) be a real number.
   Then \(x^2 \geq 0\).
   So \(x^2 + 1 > 0\).
   Hence \(\sim x^2 + 1 = 0\).
   Therefore \([[\text{there does not exist a real number } x \text{ such that } x^2 + 1 = 0}$$]

4. (optional) Prove that every year must have a Friday the 13th.

5. For each of the following proposed definitions of “prime number”, indicate whether the definition, as written, is correct or not. If it is not correct, explain why by arguing against it as in the subsection “How to argue against a definition” below.

   (NOTE: A correct definition of “prime number” is given in the book.)

   i. **DEFINITION.** A prime number is an integer that is only divisible by itself and 1. (The definition of “divisible” is the one given in these notes, a few pages below.)
ii. **Definition.** A prime number is a natural number such that the only natural numbers that divide it are 1 and $p$.

iii. **Definition.** A prime number is a natural number $p$ such that the only natural numbers that divide it are 1 and $p$.

iv. **Definition.** A prime number is a natural number such that there are exactly two natural numbers that are factors of $p$.

v. **Definition.** A prime number is a natural number $p$ such that there are exactly two natural numbers that are factors of $p$.

### 1.11 Some remarks about mathematical writing

#### 1.11.1 Write clearly in complete sentences

You should write so that you can be easily understood by a properly trained English-speaking individual. In particular, this means that you must

- Use complete English sentences, that make clearly identifiable statements with a clear meaning that can be understood by anyone reading what you wrote. For example:
  - If you tell me that “she is very smart,” but you haven’t told me who “she” is, then I don’t know who you are talking about, so you haven’t made a statement with a clear meaning.
  - If you write “$x > 0$,” but you haven’t told me who “$x$” is, then I don’t know what you are talking about, so you haven’t made a statement.
  - If I ask you to state Pythagoras’ theorem and your answer only says “$a^2 + b^2 = c^2$,” then nobody will know what you are talking about, because you have not said what “$a$,” “$b$,” and “$c$” are supposed to be.

- Avoid exaggerated or incorrect use of cryptic mathematical notation.

- Explain what you are doing.

- Make sure that letter “variables” are used correctly, that is that either:
  (i) it has been said before what these letters stand for, or (ii) they are “closed variables” (or “dummy variables,” or “bound variables”) in the sense that will be discussed in detail in class, and will also be explained later in these notes.

- Provide proper connectives between equations as well as between ideas.

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1. Of course, your teacher will know what you are trying to say, and anybody who already knows the statement of Pythagoras’ theorem will know. But when you are asked to state a theorem or a definition you should write it as if you were talking to somebody who does not know yet what the theorem or the definition say.

2. Here is a correct statement of Pythagoras’ theorem: Let $c$ be the length of the hypotenuse of a right triangle, and let $a$, $b$ be the lengths of the other two sides. Then $a^2 + b^2 = c^2$. 
• Make sure that all the rules of English grammar (including those of spelling and punctuation) are strictly obeyed. (Here are two very entertaining books about punctuation that I recommend to you: (1) *Eats, Shoots and Leaves; the Zero Tolerance Approach to Punctuation*, by Lynne Truss, (2) *Eats, Shoots and Leaves; why Commas Really Do Make a Difference!*, by Lynne Truss and Bonnie Timmons.)

• Try to say things correctly, following all the rules, but *in your own words*. Please *no rote learning*. If you have to memorize a definition or a statement, then that is not a good sign, because it indicates lack of understanding.

• Please proofread carefully what you hand in. Ideally, you should read and reread and revise almost any formal communication. **Neatness and clarity count**, as you well know if you’ve tried to read any complicated document.

• *Do not assume that the people reading your paper can read your mind*. *Do assume that they are intelligent, but also assume that they are busy, and cannot and will not spend an excessive amount of time puzzling out your meaning*. Communication is difficult, and written technical communication is close to an art.

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For the homework, effective written exposition will be worth at least 50% of your grade. Conversely, bad or unclear exposition may be penalized as much as 50% of the grade or even more. For the midterms and final exams, exposition will count for about 20% of the grade.

• The best reference known to me on effective writing is *The Elements of Style* by Strunk and White, a very thin paperback published by Macmillan. It isn’t expensive, and it is easy to read. I recommend it.

1.11.2 Your written work

You should pay attention to presentation, especially for the homework:

• A nicely typed homework (e.g., using a word processor) is preferable to handwritten work. Handwritten work is acceptable too, but in that case:
  - If you have to cross out lots of words, then you should rewrite the whole thing anew, **cleanly and neatly**. If you are not willing to spend some of your time doing this; if what you hand in shows that
you were in a hurry and that you did not make the effort to write things neatly and properly, then there is no reason for the instructor or the grader to spend any of our time reading what you wrote, and we will not do it.

- Use a pen. **Never use a pencil.**
- Use a black, blue, or green pen, but DO NOT USE RED. (Reason: The use of red is reserved for the instructor’s and grader’s comments.)
- If you tear off the sheets from a spiral notebook, please make sure before you hand them in that there are none of those ugly hanging shreds of paper at the margins. Use scissors, or a cutter, if necessary.

• Make sure that your name appears in every sheet of paper you hand in, and that if you are handing in more than one sheet then the sheets are stapled and numbered.

<table>
<thead>
<tr>
<th>If you hand in a homework assignment that has one of the following flaws:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• it is written carelessly or in a hurry,</td>
</tr>
<tr>
<td>• it has lots of words crossed out,</td>
</tr>
<tr>
<td>• it has unreadable handwriting,</td>
</tr>
<tr>
<td>• it has unstapled sheets,</td>
</tr>
<tr>
<td>• it has unnumbered sheets,</td>
</tr>
<tr>
<td>• it has sheets that fail to show your name,</td>
</tr>
<tr>
<td>• it has shreds of paper at the margins,</td>
</tr>
<tr>
<td>• it is written using pencil rather than a pen,</td>
</tr>
<tr>
<td>• it is written in red,</td>
</tr>
<tr>
<td>then you will lose points.</td>
</tr>
</tbody>
</table>

1.12 Write with precision, and avoid ambiguities

The sentences you write should be **precise**. This means that it should always be clear what you mean, and there should be **no ambiguities**.

**NOTE:** A word or phrase is **ambiguous** if it has two or more different possible meanings. Here are some examples of ambiguous phrases:

- The president promised to stop drinking on campus.
- Mary had a little lamb.
• The lady hit the man with an umbrella.
• He gave her cat food.
• In my opinion, you will be very fortunate to get this person to work for you.
• Your novel is quite unique; I have never read anything like it.

In Mathematics, there are lots of words that are or can be ambiguous. You should never use such words unless the ambiguity is resolved by the context.

Examples of ambiguous mathematical words.

• **Number.** There are lots of different kinds of “numbers”. There are real numbers, complex numbers, natural numbers, integers, integers modulo 2, integers modulo 3, etc. So *never use the word “number” unless it is clear from the context what kind of number you mean.* For example, don’t say something like “a prime number is a number that is only divisible by 1 and by itself”, because this doesn’t make it clear what kind of number you are talking about.

• **Larger than (or “greater than”).** Does this mean “>” or “≥”? It’s not clear, right? So don’t say it!!

• **Is contained in.** This is sometimes used to mean “belongs to” and sometimes to mean “is a subset of”. (We will see soon what these two expressions mean.) And these two are quite different. So *don’t use “is contained in.”*

• **Combined** (as in “2 combined with 3 . . . ”). What do you mean? Plus? Times? Something else?

1.13 Using symbols

In Mathematics, we often use symbols instead of words or sentences. There are several reasons for this, and we will discuss such reasons later, but here is one reason: **symbols are unambiguous, and having to write something using symbols forces you to eliminate ambiguities.**

**Example 1.** The symbol \( \mathbb{R} \) stands for “the set of all real numbers.” The symbol \( \mathbb{N} \) stands for “the set of all natural numbers.” The symbol \( \mathbb{Z} \) stands for “the set of all integers.” The symbol \( \mathbb{Q} \) stands for “the set of all rational numbers.” The symbol \( \in \) stands for “belongs to”, or “is a member of.” So, for example, “3 \( \in \mathbb{Z} \)” says that “3 is an integer”, and “−5 \( \in \mathbb{R} \)” says that −5 is a real number. If you want to introduce an integer and call it \( n \), you can say “let \( n \in \mathbb{Z} \)” But if you want to introduce something called \( n \) by saying “let \( n \) be a number”, and if you want to use symbolic notation, you just cannot do it, because there is no mathematical symbol for “number”, reflecting the fact that the word “number”, just like that, is vague and should not be used.
Example 2. The symbol $>$ stands for “strictly larger than” (that is, “larger than but not equal to”). The symbol $\geq$ stands for “larger than or equal to”. If you want to introduce two real numbers $x$ and $y$ and announce that $x$ is “larger than” $y$, and you want to use symbols, then you are forced to decide whether by “$x$ is larger than $y$” you mean “$x > y$” or “$x \geq y$.”

NOTE: Students often use the symbols $>$ and $\geq$ incorrectly. Be careful!

For example, a common mistake students make is to conclude, when they have two real numbers $x$, $y$, and they know it is not true that $x > y$, that $y > x$. That is not true! All you can conclude when you know that $\neg x > y$ (that is, that it’s not true that $x > y$), is that $y \geq x$.

Example 3. The symbol $\in$ stands for “belongs to”, or “is a member of”. The symbol $\subseteq$ stands for “is a subset of”. If you say something like “$x$ is contained in $y$”, then it’s not clear whether you mean “$x \in y$”, or “$x \subseteq y$”, or maybe something else. **Having to say what you want to say in symbolic language forces you to reach a more precise understanding of what you are trying to say, and to say it correctly.**

Example 4. The symbol $+$ stands for “plus”, and the symbol $\cdot$ stands for “times”. If you are thinking of saying “2 and 2 make 4,” or “2 combined with 2 is 4”, then having to say it with symbols forces you to decide whether you are trying to say that $2 + 2 = 4$ or that $2 \cdot 2 = 4$. Similarly, in set theory there are several ways to “combine” two sets $A$, $B$, namely, by taking the union $A \cup B$, the intersection $A \cap B$, or the symmetric difference $A \Delta B$. So you should write $A \cup B$, if that’s what you mean, or $A \cap B$, if that’s what you mean, or $A \Delta B$, if that’s what you mean, but never “$A$ combined with $B$.”

2 How to write definitions

A **definition** is a text consisting of one or several sentences, whose purpose is to explain exactly, with complete precision and clarity, what a given term or phrase means.

So here are some obvious requirements that every definition you write should meet:

1. It must be completely clear which is the word or phrase being defined. (Technically, this is called the **definiendum**.) This is usually achieved by **highlighting** the definiendum in some way. Our textbook uses boldface, and I recommend that you do the following experiment several times: open the book anywhere, at random, and see if you find one or more definitions. If you do, verify that each the words or phrases being defined appears in boldface. (QUESTION FOR YOU: In the first sentence of this section, on “How to write definitions”, the word “definition” is in boldface. Why?) When you write, if you do it by hand and are not using a word processor, it’s hard to do boldface, so you should **underline** instead.
In your exams, some of the questions will involve writing definitions. When you write a definition, you should underline the definiendum. If you don’t do it, you will lose points.

2. If you are defining a property, then this property will have arguments. (For example, “being a rational number” is a property of one thing, so it will have one argument. “Being divisible by” is a property of two numbers, so it will have two arguments. “Being continuous at a point” is a property of a function and a point, so this has two arguments, one a function and one a point. The arguments should be introduced before you start talking about them, and given names.)

EXAMPLES:

a. Defining “rational number”. The definition of “rational number” should be about one number, and should tell me under exactly what conditions this number is going to be said to be “rational.” So here is a correct definition of “rational number”:

**DEFINITION.** A rational number is a real number \( x \) such that there exist integers \( m, n \) for which \( n \neq 0 \) and \( x = \frac{m}{n} \).

And here is another correct way to write the same definition:

**DEFINITION.** Let \( x \in \mathbb{R} \). We say that \( x \) is a rational number (or that \( x \) is rational) if there exist integers \( m, n \) for which \( n \neq 0 \) and \( x = \frac{m}{n} \).

For comparison, here is an incorrect way to write the definition:

**DEFINITION.** A rational number is a real number such that there exist integers \( m, n \) for which \( n \neq 0 \) and \( x = \frac{m}{n} \).

(Why is this incorrect? Because you are talking about a mysterious thing called “\( x \)” but you haven’t introduced it!)

b. Defining “divides” (and the related concepts of “is divisible by”, “factor”, “multiple”). The definition of “divides” should be about two numbers (because we say that “\( x \) divides \( y \)”), and should tell us under what conditions we say that the number \( x \) divides the number \( y \). Furthermore, these numbers should be integers (or, if you prefer, natural numbers). (Why? Think! In the world of real numbers, any number “divides” any other number, as long as it is not zero. For example, 3 divides 5, because you can divide 5 by 3 and get a quotient of 5/3. So the concept of divisibility for real numbers is completely stupid.) So you should introduce your two arguments and make sure you announce that they are supposed to be integers.
Also, “$x$ divides $y$” is equivalent to “$y$ is divisible by $x$”, and to “$x$ is a factor of $y$”, and to “$y$ is a multiple of $x$”, so we can define all these four things simultaneously.

So here is a correct definition of “divides”:

**DEFINITION.** Let $a$, $b$ be integers. We say that $a$ divides $b$ if there exists an integer $k$ such that $b = ak$.

And here is another correct definition of “divides”, in which we also incorporate the definitions of ”is divisible by”, “is a factor of”, and “is a multiple of”:

**DEFINITION.** Let $a$, $b$ be integers. We say that $a$ divides $b$ (or that $b$ is divisible by $a$, or that $a$ is a factor of $b$, or that $b$ is a multiple of $a$) if there exists an integer $k$ such that $b = ak$.

And, finally, here is an *incorrect* definition:

**DEFINITION.** Let $a$, $b$ be integers. We say that $a$ divides $b$ if there exists a number $k$ such that $b = ak$.

(Why is this wrong? Because it does not tell me what kind of “number” this $k$ is supposed to be.)

### 2.1 How to argue that a definition is wrong.

One of the best ways to argue that a definition is wrong is to show that, if one accepts that definition, then something that’s clearly not true happens.

**EXAMPLE.** Suppose someone writes for you the following definition:

**DEFINITION.** A politician is a Democrat if he/she is in favor of gun control.

You can argue that this definition is wrong by saying:

According to this definition, New York City Mayor Bloomberg would be a Democrat, because he is in favor of gun control. But Mayor Bloomberg is a Republican. Therefore the definition is wrong.

Similarly, you can argue against this definition:

**DEFINITION.** Let $a$, $b$ be integers. We say that $a$ divides $b$ if there exists a number $k$ such that $b = ak$.

by saying:

According to these definition, 2 divides 3, because there exists a “number” $k$ such that $3 = 2k$, since you can take $k = 3/2$.

Of course, the author of the definition may reply with:
I didn’t mean $k$ to be a fraction! I meant $k$ to be an integer!

But then you should answer:

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How on Earth am I supposed to know that you meant $k$ to be an integer if you don’t tell me? If you meant $k$ to be an integer, you should have said so! I am not supposed to read your mind, especially in this course. You are the one who is supposed to tell me what you mean. I will not make up meanings for you; I will not assume, if you said something ambiguous, that you must have meant the right thing! Actually, whenever you say something ambiguous (in which case I have to put in the meaning for you, because you didn’t do it), then I will always put in the meaning that’s worst for you. For example, if you meant $k$ to be an integer, but didn’t tell me, then I will assume that you meant $k$ to be a real number, which will make your definition totally wrong!
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Another way to argue that a definition is wrong is to show that it is unclear and therefore meaningless. For example, if a student writes

**DEFINITION.** Let $a, b$ be integers. We say that $a$ divides $b$ if there exists a number $k$ such that $b = ak$.

(that is, the same “definition” that we discussed before), then you can argue (instead of doing what we did earlier, and interpret “number” to mean “real number”, or “rational number”, so that $3/2$ would be a “number”), that this is not clear at all, because we do not know what “number” means.

### 2.2 How to argue that a statement is wrong.

The same principles that apply for arguing that a definition is wrong work in fact for any statement. For example, suppose somebody writes the following, as one of the steps in a proof:

$$\sim (3 + \pi) \lor (3 - \pi).$$

Then you should analyze this as follows:

1. First of all, you should ask yourself: **what does this mean?** If it is a sentence, you should ask yourself: **is it true?** How do I know if it is true? **How would I find out if it is true or false?** If it is a term\(^3\), then you should ask yourself which object or entity does this stand for? **If it is neither a term nor a proposition, then it is meaningless.** And, if you cannot answer any of the questions just posed above, then it is meaningless.

\(^3\)The precise meaning of “term”, “sentence”, and “proposition” will be discussed later in these notes.
2. If the expression being considered is a composite expression, then you must look at the constituent parts, ask yourself what they mean, and then figure out how the meaning of the composite expression is derived from the simpler ones. **If you cannot do this, then the given expression is meaningless.**

Let us look, for example, at the expression

\[ \sim (3 + \pi) \lor (3 - \pi) , \]

mentioned above.

This expression is made from the symbols “3” and “\(\pi\)”, combined in some way. Now, there is no problem understanding what “3” and “\(\pi\)” mean: “3” is the name of a natural number, also known as \(2 + 1\), or \(1 + 1 + 1\). As for “\(\pi\)”, it is the name of a real number, which you have seen defined in various ways in your Calculus courses. (For example, \(\pi\) may be defined as the length of the arc of circle given by \(x^2 + y^2 = 1, y \geq 0\), and you know how to write a formula for this length, namely

\[ \pi = \int_{-1}^{1} \sqrt{1 - x^2} \, dx . \]

Or you may have seen \(\pi\) defined as the integral \(\int_{-\infty}^{\infty} \frac{dx}{1+x^2}\).

Now that we know what 3 and \(\pi\) mean, let us go on and investigate what the composite expression means. The first thing that is done with 3 and \(\pi\) is to combine them to produce the expression \(3 + \pi\). And we also combine 3 and \(\pi\) to produce the expression \(3 - \pi\). In both cases, the meanings of these combined expressions are cleear: \(3 + \pi\) and \(3 - \pi\) are names of real numbers, which are perfectly well defined, because we know how to add and subtract real numbers.

Next, the expression we are studying contains the expression “\(~ (3 + \pi)\)”, so we have to ask ourselves “what does \(~ (3 + \pi)\) mean?" Here we find ourselves unable to answer. The logical connective “\(~\)” is supposed to apply to sentences, not to terms! And \(3 + \pi\) is a term, not a sentence, so “\(~ (3 + \pi)\)” is meaningless. If you don’t believe this, just use plain common sense: you know how to negate a statement; for example, you can negate the statement “My uncle Jimmy is smart” by saying “My uncle Jimmy is not smart”, which you may represent symbolically as “\(~\)My uncle Jimmy is not smart”. And you can negate the statement “\(7 > 5\)” (i.e. “seven is larger than five”) by saying \(~ 7 > 5\) (i.e. “seven is not larger than five”). Since “\(7 > 5\)” is true, its negation (i.e. the statement \(~ 7 > 5\), or “seven is not larger than five”) is false. But you cannot negate the expression “\(3 + \pi\)” because “not \(3 + \pi\)” (or “\(~ (3 + \pi)\)” is utterly meaningless. (If you think that “not \(3 + \pi\)” means something, please tell me what it means! I am eager to find out!) When a student came to me and showed me that he had written \(~ (3 + \pi) \lor (3 - \pi)\), and I asked him “what does \(~ (3 + \pi)\) mean?”, he said “I don’t know”. And, much more seriously, when I asked him “what did you think \(~ (3 + \pi)\) meant before you came here to show it to me? he said he didn’t know either. In other words:
The student had written something without knowing what it meant! In this course, please do not do that. If you do not know the meaning of what you have written, do not write it!!!

Once you have persuaded yourself that the meaning of a particular expression is understood, then you may go on to ask yourself, if the expression in question is a statement, whether it is true. This may be difficult. For example, the sentence “every year has a Friday the 13th” turns out to be true, but proving it is not easy. And if you do not know how to prove it, or that someone else has proved, then you are not entitled to assert that it is true.

2.3 Thinking critically about what you write.

You should look at what you wrote after you have written it, and analyze it by asking yourself: “what does it mean?”, “how do I know it is true?”. If you are writing a proof, then every step you write should be a statement that you know to be true, because of one of the following reasons:

1. The statement has been given to you as one of the “assumptions”.
2. The statement is an axiom (that is, one of the statements that are the starting points of the development of the mathematical theory you are working on).
3. The statement is a definition.
4. The statement has been proved before.
5. The statement follows logically from statements that have been proved before. (Do not worry if you don’t understand what this means. We are going to spend long hours in the course explaining what it means for a statement to “follow logically” from other statements.)

You should look at what you have written asking yourself, first of all, if it is meaningful (and if it is meaningless, just cross it out and start all over again). And then you should ask whether it is true. (If a sentence isn’t true, don’t assert it.) And even that is not enough. For example, if I ask you to prove that the real number $\sqrt{2}$ is irrational, and you write

Step 1. $\sqrt{2}$ is irrational. END OF PROOF

then this is unacceptable. Why? Because, even though “$\sqrt{2}$ is irrational” is a perfectly true statement, if you are trying to prove it it means that you don’t know it is true, so you are not allowed to use it.

You may ask: if it is true, why can’t I use it?

The answer is: because the only way to know that a mathematical statement is true is to prove it, so if you don’t know how to prove it you cannot be sure it is true.
You may then ask: is it enough for me to look at the book and see that the book says that $\sqrt{2}$ is irrational?

The answer to that is: for all you know, the book could be wrong. In the world, throughout human history, there have been lots of books that were full of wrong assertions. (For example, that the Earth is flat, or that black cats bring bad luck. I leave it up to you to come with other examples.)

What I am saying here is quite simple: people and books say all kinds of things, some true, some false, some utterly meaningless. You should judge by yourself.