PRACTICE SECOND MIDTERM

Question 1. Prove that if A, B and C are any sets, then

$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$

Question 2. Let E be the relation on $\mathbb{N}$ defined by

$$xEy \quad \text{iff} \quad \text{there exist } a, b \in \mathbb{N} \text{ such that } a^2 x = b^2 y.$$

Prove that E is an equivalence relation.

(**Warning:** recall that in this course, $0 \notin \mathbb{N}$.)

Question 3. Let $\sim$ be the relation defined on $\mathbb{Z} \times \mathbb{Z}$ by

$$(a, b) \sim (c, d) \quad \text{iff} \quad ad = bc.$$

Determine whether $\sim$ is an equivalence relation.

Question 4. Let $\leq^*$ be the relation on $\mathbb{N} \times \mathbb{N}$ defined by

$$(a, b) \leq^* (c, d) \quad \text{iff} \quad a \leq c \text{ and } b \geq d.$$

(a) Prove that $\leq^*$ is a partial ordering of $\mathbb{N} \times \mathbb{N}$.

(b) Determine whether $(\mathbb{N} \times \mathbb{N}, \leq^*)$ is a linear order.

(c) Determine whether $(\mathbb{N} \times \mathbb{N}, \leq^*)$ has the least upper bound property.

Question 5.

(a) Let $\leq_1$ and $\leq_2$ be linear orderings on the set A. Let R be the relation on A defined by

$$a \, R \, b \quad \text{iff} \quad a \leq_1 b \text{ and } a \leq_2 b.$$

Prove that R is a partial order on A.

(b) Give an example of linear orderings $\leq_1$ and $\leq_2$ on a set A such that the relation R defined by

$$a \, R \, b \quad \text{iff} \quad a \leq_1 b \text{ and } a \leq_2 b.$$

is *not* a linear order.