Question 1.
(a) Give the definition of \( A \approx B \); i.e. the set \( A \) is equinumerous to the set \( B \).
(b) Prove that if \( A \) is any set, then \( A \not\approx \mathcal{P}(A) \).

Question 2.
(a) State the Schröder-Bernstein Theorem.
(b) A subset \( S \subseteq \mathbb{N} \) is a moiety if both \( S \) and \( \mathbb{N} \setminus S \) are infinite. Let
\[
\mathcal{M}(\mathbb{N}) = \{ S \subseteq \mathbb{N} \mid S \text{ is a moiety } \}.
\]
Prove that \( \mathcal{M}(\mathbb{N}) \approx \mathcal{P}(\mathbb{N}) \).
(c) Let \( \text{Fin}(\mathbb{N}) \) be the set of finite sets of natural numbers. Prove that
\[
\mathcal{P}(\mathbb{N}) \approx \mathcal{P}(\mathbb{N}) \setminus \text{Fin}(\mathbb{N}).
\]

Question 3.
(a) Define the addition operation \( \kappa + \lambda \) for cardinal numbers \( \kappa \) and \( \lambda \).
(b) Define the multiplication operation \( \kappa \cdot \lambda \) for cardinal numbers \( \kappa \) and \( \lambda \).
(c) Prove that \( \aleph_0 \cdot 2^{\aleph_0} = 2^{\aleph_0} \).
(d) Prove that if \( \kappa \) is a cardinal number such that \( \kappa > 1 \), then \( \kappa + \kappa \leq \kappa \cdot \kappa \).

Question 4.
(a) Give the definition of a well-ordering \( < \) of a set \( W \).
(b) Let \( \prec \) be the linear ordering on \( \mathbb{N} \times \mathbb{Z} \) defined by \( \langle a, b \rangle \prec \langle c, d \rangle \) if either:
- \( a < c \) or
- \( a = c \) and \( b < d \).
Determine whether \( \prec \) is a well-ordering of \( \mathbb{N} \times \mathbb{Z} \).
Question 5. Let $\prec$ be the linear ordering on $\mathbb{N} \times \mathbb{N}$ defined by $\langle a, b \rangle \prec \langle c, d \rangle$ if either:

- $\max\{a, b\} < \max\{c, d\}$, or
- $\max\{a, b\} = \max\{c, d\}$ and $a < c$, or
- $\max\{a, b\} = \max\{c, d\}$ and $a = c$ and $b < d$.

Prove that $\prec$ is a well-ordering of $\mathbb{N} \times \mathbb{N}$. [Note: You do not need to prove that $\prec$ is a linear ordering of $\mathbb{N} \times \mathbb{N}$.]