PRACTICE FIRST MIDTERM

Question 1. Negate each of the following statements.

(a) There exists a natural number n such that for every prime p, if $p > n$, then $p+2$ is not a prime.

(b) For all $\epsilon > 0$, there exist natural numbers n and m such that $|n^2 - 7m^2| < \epsilon$.

WARNING: In order to receive full credit, your final answers should be written in mathematical English. In particular, your final answers should not include abbreviations such as $\forall x$, $\exists x$, $\Rightarrow$, etc. Of course, it is acceptable (and probably a good idea) to use these abbreviations in your working.

Question 2. Use truth tables to determine whether each of the following statements is a tautology.

(a) $((P \vee Q) \Rightarrow Q) \Rightarrow P$

(b) $(P \wedge (Q \vee \sim Q)) \Leftrightarrow P$

Question 3. For each of the following statements, give either give a proof or give a counterexample.

(a) Let A and B be any sets. Then

$$\mathcal{P}(A) \smallsetminus \mathcal{P}(B) \subseteq \mathcal{P}(A \smallsetminus B).$$

(b) If A, B and C are any sets, then

$$(A \smallsetminus B) \smallsetminus C = (A \smallsetminus C) \smallsetminus (B \smallsetminus C)$$

Question 4. Suppose that A, B and C are sets which satisfy both of the following conditions:

- $A \cup C = B \cup C$; and
- $A \cap C = B \cap C$.

Prove that $A = B$.

Question 5. Prove that $10^{n+1} + 3 \cdot 4^{n-1} + 5$ is divisible by 9 for all $n \geq 1$.

Question 6. The Fibonacci sequence is defined recursively by

$$c_0 = 1$$

$$c_1 = 1$$

$$c_{n+2} = c_{n+1} + c_n$$

Prove by induction that for all $n \geq 0$,

$$c_0 + c_3 + c_6 + \cdots + c_{3n} = \frac{c_{3n+2}}{2}.$$