

## HOMEWORK 9

**Definition.** Two sets  $A$  and  $B$  are *equinumerous*, written  $A \approx B$ , iff there exists a bijection  $f : A \rightarrow B$ .

**Question 1.** Suppose that  $A$ ,  $B$  and  $C$  are any sets. Prove:

- (i)  $A \approx A$ .
- (ii) If  $A \approx B$ , then  $B \approx A$ .
- (iii) If  $A \approx B$  and  $B \approx C$ , then  $A \approx C$ .

**Question 2.** Prove or disprove the following statements.

- (a) Suppose that  $A$  and  $B$  are any sets. If  $A \approx B$  and  $f : A \rightarrow B$  is an injection, then  $f$  is a bijection.
- (b) Suppose that  $A$  and  $B$  are any sets. If  $A \approx B$  and  $f : A \rightarrow B$  is a surjection, then  $f$  is a bijection.

**Question 3.** Let  $f : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$  be the function defined by

$$f(m, n) = 2^{m-1}(2n - 1).$$

- (a) Prove that  $f$  is an injection.
- (b) Prove that  $f$  is a surjection.