HOMEWORK 8

Question 1. Let $\leq^*$ be the relation on $\mathbb{Q} \times \mathbb{Q}$ defined by

$$(a, b) \leq^* (c, d) \quad \text{iff} \quad a \leq c \text{ and } b \geq d.$$

(a) Prove that $\leq^*$ is a partial ordering of $\mathbb{Q} \times \mathbb{Q}$.

(b) Determine whether $(\mathbb{Q} \times \mathbb{Q}, \leq^*)$ has the least upper bound property.

(*Hint:* You may make use of the fact that $\sqrt{2} \notin \mathbb{Q}$.)

Question 2. Suppose that R is a relation on the set A which satisfies *both* of the following conditions:

(i) R is an equivalence relation on A; and

(ii) R is a partial ordering of A.

Prove that for all $a, b \in A$,

$$a \, R \, b \quad \text{iff} \quad a = b.$$