# HOMEWORK 7

**Question 1.** Let $E_1$ be an equivalence relation on the set $A_1$ and let $E_2$ be an equivalence relation on the set $A_2$. Let $R$ be the relation on the set $A_1 \times A_2$ defined by

$$(a_1, a_2)\, R\, (b_1, b_2) \quad \text{iff} \quad a_1\, E_1\, b_1 \text{ and } a_2\, E_2\, b_2.$$

Prove that $R$ is an equivalence relation on $A_1 \times A_2$.

**Question 2.** Let $R$ be the relation on $\mathbb{N} \times \mathbb{N}$ defined by

$$(a, b)\, R\, (c, d) \quad \text{iff} \quad a \leq c \text{ or } b \leq d.$$

Determine whether $R$ is a partial ordering of $\mathbb{N} \times \mathbb{N}$.

**Question 3.** Let $\mid$ be the partial ordering of $\mathbb{N}$ defined by $a \mid b$ iff $a$ is a (not necessarily proper) divisor of $b$. For each of the following statements, either give a proof or give a counterexample.

(a) In the poset $(\mathbb{N}, |)$, each element has infinitely many immediate successors.

(b) If $\emptyset \neq K \subseteq \mathbb{N}$, then $K$ is bounded above iff $K$ is a finite subset.