HOMEWORK 6

Question 1. Prove that if A, B and C are any sets, then

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

Question 2. (a) Prove that if A, B, C and D are any sets, then

$$(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D).$$

(b) Give an example of sets A, B, C and D such that

$$(A \times B) \cup (C \times D) \neq (A \cup C) \times (B \cup D).$$

Question 3. Let

$$S = \{(a, b) \mid a, b \in \mathbb{Z}, b \neq 0\}$$

and let $\sim$ be the relation defined on S by

$$(a, b) \sim (c, d) \quad \text{iff } ad = bc.$$

Prove that $\sim$ is an equivalence relation.

Question 4. Let R be the relation on $\mathbb{R} \setminus \{0\}$ defined by

$$a \, R \, b \quad \text{iff} \quad \frac{a}{b} \in \mathbb{Q} \text{ or } a - b \in \mathbb{Q}.$$

Determine whether R is an equivalence relation.
(*Hint:* You may make use of the fact that $\sqrt{2} \notin \mathbb{Q}$.)