HOMEWORK 5

Question 1. Prove by induction that for all $n \in \mathbb{N}$,

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

Question 2. Consider the sequence defined recursively by

$$a_1 = 2$$

$$a_{n+1} = 7a_n + 9^n + 5^n$$

Prove by induction that for all $n \geq 1$,

$$a_n = \frac{9^n - 5^n}{2}.$$

Question 3. Consider the sequence defined recursively by

$$a_1 = 1$$

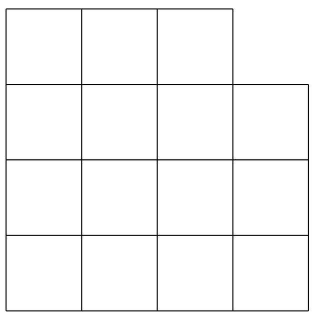
$$a_2 = 3$$

$$a_{n+2} = 3a_{n+1} - 2a_n$$

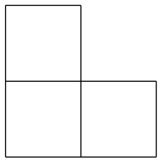
Prove by induction that for all $n \geq 1$,

$$a_n = 2^n - 1.$$

Question 4. Consider the following 4×4 square grid from which one square has been removed:



Then it is easily checked that it can be covered without overlaps using L-shaped tiles of the following form:



Prove that for any $n \geq 1$, a $2^n \times 2^n$ square grid with *any* one square removed can be covered without overlaps using such L-shaped tiles.