HOMEWORK 4

Question 1. Prove by induction that for all $n \in \mathbb{N}$,

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

Question 2. Let $x \neq 1$ be a real number. Prove by induction that for all $n \in \mathbb{N}$,

$$1 + x + x^2 + \cdots + x^{n-1} + x^n = \frac{x^{n+1} - 1}{x - 1}.$$

Question 3. Prove by induction that for all $n \geq 1$,

$$\sum_{i=1}^{n} \frac{1}{(i+2)(i+3)} = \frac{1}{3} - \frac{1}{n+3}$$

Question 4. Suppose that $A_1, \ldots, A_n$ is a list of sets such that for all i, j with $1 \leq i, j \leq n$, we have either $A_i \subseteq A_j$ or $A_j \subseteq A_i$. Prove that there exists an integer k with $1 \leq k \leq n$ such that $A_k \subseteq A_i$ for all i with $1 \leq i \leq n$. (*Hint:* Argue by induction on $n \geq 1$.)