Question 1. If $R$ is a Noetherian ring and $\varphi : R \to S$ is a surjective ring homomorphism, then $S$ is also Noetherian.

Question 2. If $R$ is a Noetherian ring and $T$ is a multiplicative subset of $R$, then the ring $T^{-1}R$ is also Noetherian.

Question 3. Let $R$ be a Noetherian ring and let $\varphi : R \to R$ be a ring homomorphism. Prove that if $\varphi$ is surjective, then $\varphi$ is an automorphism of $R$.

Question 4. Let $I$ be the ideal of $\mathbb{Z}[x]$ defined by

$$I = (2^n, 2^{n-1}x, \ldots, 2x^{n-1}, x^n).$$

Prove that $I$ cannot be generated by fewer than $n + 1$ elements.