HOMEWORK 8

Question 1. If R is a Noetherian ring and $\varphi : R \to S$ is a surjective ring homomorphism, then S is also Noetherian.

Question 2. If R is a Noetherian ring and T is a multiplicative subset of R, then the ring $T^{-1}R$ is also Noetherian.

Question 3. Let R be a Noetherian ring and let $\varphi : R \to R$ be a ring homomorphism. Prove that if φ is surjective, then φ is an automorphism of R.

Question 4. Let I be the ideal of $\mathbb{Z}[x]$ defined by

$$I = (\, 2^n, 2^{n-1}x, \cdots, 2x^{n-1}, x^n \,).$$

Prove that I cannot be generated by fewer than $n+1$ elements.