HOMEWORK 1

Question 1. Suppose that $n \geq 5$ and that

$$H = \{\pi \in \mathrm{Sym}(n) \mid \pi[\{1,2\}] = \{1,2\}\}.$$

Prove that H is a maximal subgroup of $\mathrm{Sym}(n)$.

In particular, $\mathrm{Sym}(5)$ *has a maximal subgroup of index 10.*

Question 2. Suppose that the field F is an algebraic extension of the field K. Prove that if R is a subring of F such that $K \subseteq R \subseteq F$, then R is a field.

Question 3. Let $\mathbb{Q}^{alg}$ be the algebraic closure of $\mathbb{Q}$.

(a) Prove that $[\mathbb{Q}^{alg} : \mathbb{Q}] = \aleph_0$.

(b) Prove that $|\mathrm{Aut}(\mathbb{Q}^{alg})| = 2^{\aleph_0}$.