HOMEWORK 1

**Question 1.** Suppose that $n \geq 5$ and that
\[ H = \{ \pi \in \text{Sym}(n) \mid \pi([1, 2]) = \{1, 2\} \}. \]

Prove that $H$ is a maximal subgroup of $\text{Sym}(n)$.

*In particular, $\text{Sym}(5)$ has a maximal subgroup of index 10.*

**Question 2.** Suppose that the field $F$ is an algebraic extension of the field $K$.

Prove that if $R$ is a subring of $F$ such that $K \subseteq R \subseteq F$, then $R$ is a field.

**Question 3.** Let $\mathbb{Q}^{alg}$ be the algebraic closure of $\mathbb{Q}$.

(a) Prove that $[\mathbb{Q}^{alg} : \mathbb{Q}] = \aleph_0$.

(b) Prove that $|\text{Aut}(\mathbb{Q}^{alg})| = 2^{\aleph_0}$. 