PRACTICE FIRST MID-TERM QUESTIONS

Question 1. Prove that if $A$ is any set, then $A \not\sim \mathcal{P}(A)$.

Question 2. (a) State the Cantor-Bernstein Theorem.
   (b) Prove that $\mathbb{N} \sim \mathbb{Z}[1/2]$, where $\mathbb{Z}[1/2] = \{m/2^n \mid m \in \mathbb{Z} \text{ and } n \in \mathbb{N}\}$.
   (c) Let $\text{Surj}(\mathbb{N})$ be the set of surjective functions $f : \mathbb{N} \to \mathbb{N}$. Prove that $\mathcal{P}(\mathbb{N}) \sim \text{Surj}(\mathbb{N})$.

Question 3. (a) Give the definition of a linear ordering.
   (b) Give the definition of an isomorphism between two linear orders $\langle L_1, \prec \rangle$ and $\langle L_2, \bowtie \rangle$.
   Prove or disprove each of the following statements.
   (c) $\langle \mathbb{Q} \setminus (0, 1), \prec \rangle \simeq \langle \mathbb{Q} \setminus [2, 3], \prec \rangle$.
   (d) $\langle \mathbb{Q} \setminus \mathbb{Z}, \prec \rangle \simeq \langle \mathbb{Q}, \prec \rangle$.
   (e) $\langle \mathbb{Z}[1/2], \prec \rangle \simeq \langle \mathbb{R} \setminus \mathbb{N}, \prec \rangle$, where $\mathbb{Z}[1/2] = \{m/2^n \mid m \in \mathbb{Z} \text{ and } n \in \mathbb{N}\}$.

Question 4. Determine whether each of the following wffs is a tautology.
   (a) $(A \to (B \to (A \leftrightarrow B)))$
   (b) $((P \land Q) \to (P \to Q))$

Question 5. Suppose that $\alpha$ is a wff which only involves the connectives $\{\land, \lor\}$ and the sentence symbols $\{A_1, \cdots, A_n\}$. Prove that if $\nu$ is a truth assignment such that

$$\nu(A_1) = \nu(A_2) = \cdots = \nu(A_n) = T,$$

then $\nu$ satisfies $\alpha$.

(Hint: Argue by induction on the length of the wff $\alpha$.)