MATH 350 HOMEWORK: FALL 2019

All of the homework exercises are taken from the course textbook:
Linear Algebra (5th Edition) by Friedberg, Insel and Spence.

1. First Week

As the bookstore does not yet have the 5th edition in stock, this week (and this week only) I will write out the homework exercises.

Exercise 1.2.18: Let $V = \{(a_1, a_2) \mid a_1, a_2 \in \mathbb{R}\}$. For $(a_1, a_2), (b_1, b_2) \in V$ and $c \in \mathbb{R}$, define

$$ (a_1, a_2) + (b_1, b_2) = (a_1 + 2b_1, a_2 + 3b_2) $$

and

$$ c(a_1, a_2) = (ca_1, ca_2). $$

Is $V$ a vector space over $\mathbb{R}$ with these operations? Justify your answer.

Exercise 1.2.19: Let $V = \{(a_1, a_2) \mid a_1, a_2 \in \mathbb{R}\}$. Define addition of elements of $V$ coordinatewise, and for $(a_1, a_2) \in V$ and $c \in \mathbb{R}$, define

$$ c(a_1, a_2) = \begin{cases} (0, 0) & \text{if } c = 0; \\ (ca_1, \frac{a_2}{c}) & \text{if } c \neq 0. \end{cases} $$

Is $V$ a vector space over $\mathbb{R}$ with these operations? Justify your answer.

Exercise 1.3.8: Determine whether the following sets are subspaces of $\mathbb{R}^3$ under the usual operations of vector addition and scalar multiplication. Justify your answers.

(a) $W_1 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 \mid a_1 = 3a_2 \text{ and } a_3 = -a_2 \}.$

(f) $W_6 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 \mid 5a_1^2 - 3a_2^2 + 6a_3^2 = 0 \}.$
2. THE REMAINDER OF THE SEMESTER

Unless otherwise noted, the numbering of the homework exercises is identical in the 4th and 5th editions of the textbook.

Week 2: 1.4.3(f), 1.4.5(h), 1.4.15, 1.6.7

Week 3: 1.6.14, 2.1.3, 2.1.4
In Exercises 2.1.3 and 2.1.4, you are only required to find bases of $N(T)$ and $R(T)$.

Week 4:
- 5th Edition: 2.2.3, 2.2.17
- 4th Edition: 2.2.3, 2.2.16

Hint: In the second exercise, it is helpful to think about the proof of the Dimension Theorem.

Week 6:
- 2.5.6(c), 2.5.6(d), 3.2.19, 3.2.21, 3.2.22

Week 7:
- 3.2.5(g), 3.2.5(h), 3.3.7(c), 3.3.7(e)
- Prove that if $A, B \in M_{n \times n}(F)$ and $AB = I_n$, then $BA = I_n$.

Week 8:
- 3.4.2(h), 3.4.2(i), 4.2.15, 4.2.18, 4.3.10

Week 9:
- 4th Edition: 5.1.3(d), 5.1.8(a), 5.1.8(b), 5.2.2(d), 5.2.2(e)
- 5th Edition: 5.1.4(d), 5.1.9(a), 5.1.9(b), 5.2.2(d), 5.2.2(e)

Week 10:
- 7.1.2(c), 7.1.2(d), 7.2.4(a)