Due date: February 21 (in the beginning of the class period)

1. (20 points)
   (a) Find integers $x$ and $y$ such that $13x + 101y = 1$. Use the extended Euclidean algorithm to do so, and be sure to show your work.
   (b) Find $13^{-1} \pmod{101}$.

2. (20 points) Use the Euclidean algorithm to compute $\gcd(78192, 55931)$. Also find integers $x$ and $y$ such that $78192x + 55931y = \gcd(78192, 55931)$. (Be sure to show your work.)

3. (20 points) Compute $\gcd(4883, 4369)$. Also factor 4883 and 4369 into products of primes. (Taken from Washington-Trappe 3.13.5)

4. (20 points) Compute the multiplicative inverse of 743 in $\mathbb{Z}/10037\mathbb{Z}$. (Be sure to show your work.)

5. (20 points) Let $m \geq 1$ be an integer and suppose that $a, b, c, d, k$ are integers such that $a \equiv b \pmod{m}$, $c \equiv d \pmod{m}$ and $k \geq 0$. Then show that for all integers $x, y$

   (a) $a + c \equiv b + d \pmod{m}$
   (b) $a \times d \equiv b \times c \pmod{m}$
   (c) $ax + cy \equiv bx + dy \pmod{m}$
   (d) $a^k \equiv b^k \pmod{m}$