Due date: Nov 4 (in class)
Collaboration is encouraged. However the writeup should be your own. You should cite all collaborators and any other references that you might be using.

1. Show that if P = NP, then EXP = NEXP.

2. In class we showed that the class NL can (alternatively) be defined as the class of languages $L$ that are decidable by a deterministic log-space Turing machine $M$ (i.e the work tape can only use logarithmic space) with one extra read-once tape (whose head only allowed to stay in place or move to the right). The read-once tape is allowed to contain a polynomial length witness, and an input $x \in L$ if and only if there is some witness that can make $M$ accept. Show that if the read-once tape is replaced by a tape on which the head can move both ways (but is still read-only), then such Turing machines can decide any language in NP.

3. Describe a language $L$ that is decidable, and is contained in $P/poly$, but is not contained in $P$.

4. We say that a language $L \subseteq \{0,1\}^*$ is sparse if there is some polynomial $p : \mathbb{N} \to \mathbb{N}$ such that for all $n$, $|L \cap \{0,1\}^n| \leq p(n)$. Show that if a sparse language is NP-complete then the polynomial hierarchy collapses.

5. State a complete problem for each level of the polynomial hierarchy, and sketch a proof that it is complete under polynomial time reductions. Also show that if for any $i$, $\sum_p^p = \prod_p^p$, then the polynomial hierarchy collapses to the $i$th level.

6. Let ZPP be the class of languages $L$ that can be decided by a probabilistic Turing machine $M$ that halts on each input in expected polynomial time, and for each input $x$, $\Pr[M(x) = L(x)] = 1$. Prove that $ZPP = RP \cap \text{co-RP}$.