Due date: October 7 (in class)

Collaboration is encouraged. However the writeup should be your own. You should cite all collaborators and any other references that you might be using.

1. A 2-dimensional Turing machine is one in which each tape is an infinite 2-dimensional grid where the tape can move up and down in addition to right and left. Show that if a function $f$ can be computed by a 2-dimensional Turing machine in time $T(n)$ for inputs of length $n$ (for some time-constructible function $T$), then $f$ can be computed by a usual (with one-dimensional tapes) Turing machine in time $T(n)^2$.

2. Give short proofs/justifications for the following:
   
   (a) Show that the complement of every NP-complete language is coNP-complete.
   
   (b) Show that the union of two languages in NP is also in NP. How about the intersection? What about the union of two languages in co-NP?
   
   (c) For a string $x \in \{0, 1\}^*$, let $\overline{x}$ be the complemented string where every 0 is replaced with a 1 and every 1 is replaced with a 0. For a language $L$ in NP, let $L^*$ be the language such that $x \in L$ if and only if $\overline{x} \in L^*$. What can be said about $L^*$? Is it in NP? Is it in coNP?
   
   (d) Show that the language HALT (as described in class) is NP-hard. Is it NP complete? (HALT is the set of all pairs $(M, x)$ where $M$ is a Turing machine and $x$ is a string such that $M$ halts on input $x$).

3. Show that if $P = NP$, then one can decide whether or not two graphs are isomorphic in polynomial time.

4. Show that if $P = NP$, then one can in polynomial time find a satisfying assignment to any 3-CNF formula.

5. NEXP is the class of languages decided by a nondeterministic Turing machine in exponential time. We say a language is NEXP-complete if it is in NEXP and every language in NEXP is polynomial time reducible to it. Describe an NEXP complete language $L$ and show that if $L \in EXP$ then NEXP = EXP.

6. Let QUADEQ be the language of satisfiable systems of quadratic equations over $\mathbb{F}_2$, where a quadratic equation over $\mathbb{F}_2$ is an equation of the form $\sum_{i,j} a_{ij}x_ix_j = b$, with all addition/multiplication being modulo 2. Show that QUADEQ is NP-complete.