Homework-2 (Computational Complexity)

Due date: October 30 (in class)
Collaboration is encouraged. However the writeup should be your own. You should cite all collaborators and any other references that you might be using.

1. In class we showed that the class NL can (alternatively) be defined as the class of languages $L$ that are decidable by a deterministic log-space Turing machine $M$ (i.e the work tape can only use logarithmic space) with one extra read-once tape (whose head only allowed to stay in place or move to the right). The read-once tape is allowed to contain a polynomial length witness, and an input $x \in L$ if and only if there is some witness that can make $M$ accept. Show that if the read-once tape is replaced by a tape on which the head can move both ways (but is still read-only), then such Turing machines can decide any language in NP.

2. Describe a language $L$ that is decidable, and is contained in $P/poly$, but is not contained in $P$.

3. We say that a language $L \subseteq \{0,1\}^*$ is sparse if there is some polynomial $p : \mathbb{N} \to \mathbb{N}$ such that for all $n$, $|L \cap \{0,1\}^n| \leq p(n)$. Show that if a sparse language is NP-complete then the polynomial hierarchy collapses.

4. State a complete problem for each level of the polynomial hierarchy, and sketch a proof that it is complete under polynomial time reductions. Also show that if for any $i$, $\sum_i^p = \prod_i^p$, then the polynomial hierarchy collapses to the $i$th level.

5. Let $ZPP$ be the class of languages $L$ that can be decided by a probabilistic Turing machine $M$ that halts on each input in expected polynomial time, and for each input $x$, $\Pr[M(x) = L(x)] = 1$. Prove that $ZPP = RP \cap \text{co-RP}$. 